2D deformable models from 3D data

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1 Summary of Previous and Current Work

I have been working on different topics (e.g. mobile robotics, gesture recognition) but lately I have been focused on building 2-D models from 3-D data, by means of non-rigid extensions of Procrustes Analysis (PA) illustrated in Fig. 1.

PA has been a popular technique to align and build 2-D statistical models of shapes. Given a set of shapes PA is applied to remove rigid transformations. Later, a non-rigid 2-D model is computed by applying Principal Component Analysis (PCA) to the residual. Although PA has been widely used, it has several limitations. First, the alignment to remove rigid transformations and PCA are computed independently, which can result in a poor model. Moreover, the 2-D shapes are not necessarily uniformly sampled across possible views of the 3-D object of interest, and might result in a biased model. Finally, PA scales linearly with the number of samples and landmarks and quadratically with the dimension of the data.

Due to advances in 3-D capture systems, nowadays it is common to have access to 3-D shape models for a variety of objects. If n 3-D shapes are given, we can compute r projections at uniformly sampled angles for each one of them, and learn a 2-D model from the enhanced 2-D dataset of rn shapes. However, it is difficult to generate uniform distributions in the Special Orthogonal group $SO(3)$, when the rotation matrices must be confined in a specific region $\mathbf{\Omega}$ of $SO(3)$, restricted by rotation angles $\boldsymbol{\omega} = \{\phi, \theta, \psi\}$. Moreover, the computational complexity of building the model from the enhanced dataset increases linearly with the number of projections (r).

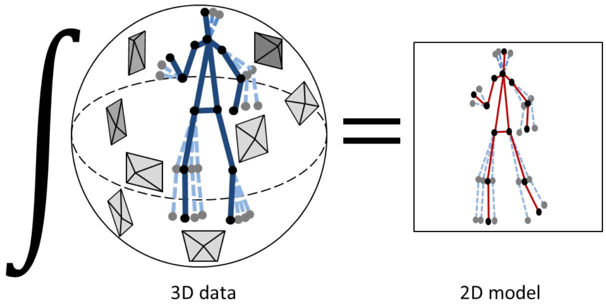


Figure 1: Illustration of the Continuous Subspace Procrustes Analysis (CSPA), which builds an unbiased 2-D model (b) by integrating over all possible viewpoints of a 3-D data set (a).

In order to deal with these drawbacks, Continuous Procrustes Analysis (CPA) was proposed in [1] by formulating PA with a functional analysis framework:

$$E_1 = \sum_{i=1}^{n} \int_{\mathbf{\Omega}} \|\mathbf{P}(\boldsymbol{\omega})\mathbf{D}_i - \mathbf{A}(\boldsymbol{\omega})_i \mathbf{M}\|_F^2 \, d\boldsymbol{\omega}, \quad (1)$$

where $\mathbf{M}_i \in \mathbb{R}^{2\times\ell}$ is the 2-D mean shape, $\mathbf{D}_i \in \mathbb{R}^{3\times\ell}$ is the 3-D ith shape composed by ℓ landmarks, $\mathbf{P}(\boldsymbol{\omega}) \in \mathbb{R}^{2\times 3}$ is an orthographic projection of a 3-D rotation $\mathbf{R}(\boldsymbol{\omega})$ in a given domain $\mathbf{\Omega}$, $\mathbf{A}(\boldsymbol{\omega})_i \in \mathbb{R}^{2\times 2}$ is an affine transformation mapping $\mathbf{M}$ to the projected data, and $d\boldsymbol{\omega} = \frac{1}{8\pi^2}\sin(\theta)d\phi d\theta d\psi$ ensures uniformity in $SO(3)$.

This continuous formulation finds the optimal 2-D projected mean shape of a 3-D dataset in a given rotation domain, by integrating over all possible rotations in that domain. It is important to notice that the 2-D projections are not explicitly computed, being an approach extremely efficient in space and time.

However, CPA only aligns the data w.r.t the mean. Later PCA should be computed in the residual error to model non-rigid deformations. However, these two steps are not optimal because they are independently computed. In addition, the PCA step would require to enhance the dataset with the 2-D projections of the 3-D data.

We recently proposed Subspace Procrustes Analysis (SPA) to address these issues. Given several instances of a 3-D object, SPA computes a 2-D mean and a subspace that can model all rigid and non-rigid deformations of the 3-D object, as illustrated in Fig. 1. We proposed a discrete (DSPA) and a continuous (CSPA) formulations for SPA. CSPA is more efficient in space and time, and produces unbiased 2-D models by minimizing:

$$E_2 = \sum_{i=1}^{n} \int_{\mathbf{\Omega}} \|\mathbf{P}(\boldsymbol{\omega})\mathbf{D}_i - \mathbf{A}(\boldsymbol{\omega})_i\mathbf{M} - (\mathbf{c}(\boldsymbol{\omega})_i \otimes \mathbf{I}_2)\mathbf{B}\|_F^2 \, d\boldsymbol{\omega}, \tag{2}$$

where $\mathbf{B} \in \mathbb{R}^{2k\times\ell}$ and $\mathbf{c}(\boldsymbol{\omega})_i \in \mathbb{R}^{k\times 1}$ are the k basis of the 2-D subspace and their weights, respectively, modeling the non rigid deformations of the projected data.

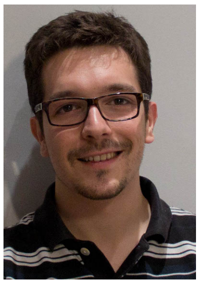
2 Future Work and Challenges

Our future work will evolve in three main directions. First, the study of new discrete and continuous formulations, as well as optimization process, to avoid unbiased models. Second, the integration not only over the rotation domain but also over the non-rigid deformations of the dataset. Finally, we will use our unbiased models in the field of human pose estimation [2].

Publications

[1] Igual, L., Perez-Sala, X., Escalera, S., Angulo, C. and De la Torre, F.,Continuous Generalized Procrustes analysis. In *Pattern Recognition*, 47(2), 659–671, 2014.

[2] Perez-Sala, X., Escalera, S., Angulo, C. and Gonzàlez, J., A survey on model based approaches for 2D and 3D visual human pose recovery. In *Sensors*, 14(3), 4189–4210, 2014.



Xavier Prez-Sala received the B.Sc. degree in Industrial Electronics (2008) and the M.Sc. degree in Artificial Intelligence (2010) from the Technical University of Catalonia (UPC). He did his master thesis combining computer vision and machine learning techniques for social robots navigation. He is currently a research member at Fundació Privada Sant Antoni Abat and a Ph.D. student at UPC. His research is mainly focused on human pose estimation, in the field of social robotics.