The inner equation of generic analytic unfoldings of the Hopf-zero singularity

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Abstract

We consider the family of differential equations given by

$$\begin{aligned} \frac{d\phi}{d\tau} &= \eta\phi - (\alpha + c\eta)\,\mathrm{i}\phi + \varepsilon F_1(\phi,\varphi,\eta) \\ \frac{d\varphi}{d\tau} &= \eta\varphi + (\alpha + c\eta)\,\mathrm{i}\varphi + \varepsilon F_2(\phi,\varphi,\phi) \\ \frac{d\eta}{d\tau} &= \eta^2 + b\varphi\phi + \varepsilon H(\phi,\varphi,\eta) \end{aligned} \tag{1}$$

where $(F_1, F_2, H)(\phi,\varphi,\eta) = O(\|(\phi,\varphi,\eta)\|^3)$ are analytic functions in $B(r_0) := \{\zeta \in \mathbb{C}^3 : \|\zeta\| < r_0\}$ for some $r_0 > 0$. The parameter ε is not necessarily small.

After some change of variables, system (1) can be seen as the inner equation of the generic analytic unfoldings of the Hopf-zero singularity.

We will prove that system (1) has solutions $\Psi^\pm = (\phi^\pm, \varphi^\pm, \eta^\pm)$ satisfying the asymptotic condition

$$\lim_{\mathrm{Re}\tau \to \pm\infty} \Psi^\pm(\tau,\varepsilon) = 0.$$

Let $\Delta\Psi = \Psi^- - \Psi^+$. We will also check that there exists an analytic function $C(\varepsilon)$ such that

$$\begin{pmatrix} \pi^{1,2}\Delta\Psi(\tau,\varepsilon) \\ \tau^4\pi^3\Delta\Psi(\tau,\varepsilon) \end{pmatrix} \sim \tau \mathrm{e}^{-\mathrm{i}(|\alpha|\tau - c\log\tau)} C(\varepsilon), \qquad \mathrm{Im}\ \tau \to -\infty.$$