Gevrey- and analytic models for families of vector fields

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Abstract

We look for local simplified forms for equilibria of families of vector fields X_λ. If the family is Gevrey-s in the space variable, with $0 \leq s \leq 1$, we look for a family of Gevrey-s changes of variables putting the system in a form as simple as possible. This includes the analytic case (i.e. $s = 0$). The dependence on the parameter is also studied, according to the case. An example is the classical result of Poincaré for a hyperbolic source or sink: if X_λ is analytic (also in the parameter λ) then there is a local analytic conjugacy with a polynomial form with coefficients that are analytic in λ. For a saddle one cannot hope for such a type of result in the presence of parameters. In some applications one must consider divergent series in the Gevrey category, for example when studying the center manifold. We give sufficient conditions on the spectrum at the equilibrium and on the simplified form such that a Gevrey-s family can be Gevrey-s conjugated to the simplified form. Note that analyticity implies Gevrey-s for $s \geq 0$. As a special case we obtain that certain center manifolds of analytic vector fields are of Gevrey-1 type. The Poincaré theorem just mentioned is also a special case.

We also study the asymptotic properties of the conjugacy on a polysector with opening angles smaller than $s\pi$ in $\mathbf{C}^n$ by considering a Borel-Laplace resummation.