Existence of diffusion in a-priori unstable models: higher order resonances do not impede diffusion

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Abstract

Let $I_i \in R, \phi_i \in T^n$ be action angle variables, for $i = 1, N$ $p_j \in R$, $q_i \in T^n$ be action angle variables for $j = 1, d$.

We show Arnold diffusion in models of the form:

$$H(I_1, \dots, I_N, \phi_1, \dots, p_1, \dots p_d, q_1, \dots q_d, t) =$$

$$h(I_1, \dots I_d) + \sum_j \pm p_2^i + V_i(q_i) + \epsilon P((I_1, \dots, I_N, \phi_1, \dots, p_1, \dots p_d, q_1, \dots q_d, t)$$

More precisely, we show that if P is a trigonometric polynomial in ϕ, q, t and that the system satisfies some generic non-degeneracy assumptions, then, given any generic curve $I = \gamma(s)$, then there exist orbits $x(t)$ of the system above and time reparameterizations – i.e. increasing functions of r – such that $I(x(t)) - \gamma(r(t)| \le \epsilon^{1/2}$.

The genericity conditions on the h, V, P are very explicit and can be verified in concrete systems. The non-degeneracy assumptions in γ are that the curve γ avoids the resonances of order greater than 2 and that it transverses the first rank resonant surfaces (i.e. the regions where the modulus of resonant vectors is 1-dimensional) of order $1, 2$ transversally.

The method of proof is an extension of the method of [DLS06]. We show that a KAM whiskered tori outside of resonant regions, has heteroclinic connections with all the KAM tori in a neighborhood. In neighborhoods of first and second order resonant regions, we show that the method of [DLS06] applies and we can transverse the region. The regions of rank one resonaces can be transversed by the method of [DLS06] provided that the intersection point is far from rank 2 resonances. Fortunately, the set of rank 2 resonances is of codimension 2 in the action space and can be avoided by the diffusion path.