High-precision computations of divergent asymptotic series and homoclinic phenomena

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Abstract

We study asymptotic expansions for the exponentially small splitting of separatrices both from the analytical and numerical point of view. Using analytic information, we can determine asymptotic coefficients numerically. The computations are performed with high-precision arithmetic, which involves up to several thousands digits. This approach allows us to obtain information, which is usually considered to be out of reach of numerical methods.

We use our results to test Borel-Laplace summability of the asymptotic series and to study positions and types of singularities on the Borel plane. In particular, we consider generalisations of the standard map, Hamiltonian dynamical systems near a strong resonance, and the discrete Bogdanov-Takens bifurcation.

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