On a homoclinic origination of Hénon maps.

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Abstract

Hénon maps are called quadratic maps with constant Jacobian whose inverse are quadratic again. Any two-dimensional quadratic map with constant Jacobian can be brought to the standard form $\bar{x} = y, \bar{y} = M_1 - Bx - y^2$, where M_1 and B are parameters (B is the Jacobian). The main standard form for three-dimensional Hénon maps looks as [1, 2]

$$\bar{x} = y, \bar{y} = z, \bar{z} = M_1 + M_2^1 y + M_2^2 z + Bx + ay^2 + byz + cz^2,$$

where one of the parameters M_2^1 or M_2^2 can be nullified (when $|a| + |c| \neq 0$) and a normalization condition, like $|a| + |b| + |c| = 1$, is fulfilled. There are other two degenerate forms for 3D Hénon maps (see [1]).

Note that these Hénon maps (and their generalizations) can appear near homoclinic tangencies as rescaled return maps. Moreover, the dynamics of these maps can be quite non-trivial. Thus, one of the recent results in this direction – the 3D Henon map with $a = b = M_2^1 = 0$ can possess Lorenz-like wild strange attractors – was established in our joint with C.Simó paper [3].

References

[1] H.E. Lomelí and J.D. Meiss. *Nonlinearity*, 11:557–574, 1998.

[2] S.V.Gonchenko, J.D.Meiss, I.I.Ovsyannikov "Chaotic dynamics of three-dimensional Hénon maps that originate from a homoclinic bifurcations".- Reg. and Chaotic Dyn., 2006 (to appear).

[3] S.V. Gonchenko, I.I. Ovsyannikov, C. Simó, and D.V. Turaev. *Int. J. of Bifurcation and Chaos*, 15(11), 2005, 3493-3508.