Bifurcations of stationary measures of random diffeomorphisms

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Abstract

Random diffeomorphisms with bounded absolutely continuous noise are known to possess a finite number of stationary measures. We discuss dependence of stationary measures on an auxiliary parameter, thus describing bifurcations of families of random diffeomorphisms. A bifurcation theory is developed under mild regularity assumptions on the diffeomorphisms and the noise distribution (e.g. smooth diffeomorphisms with uniformly distributed additive noise are included). We distinguish bifurcations where the density function of a stationary measure varies discontinuously or where the support of a stationary measure varies discontinuously.

We establish that generic random diffeomorphisms are stable. Densities of stable stationary measures are shown to be smooth and to depend smoothly on an auxiliary parameter, except at bifurcation values. The bifurcation theory explains the occurrence of transients and intermittency as the main bifurcation phenomena in random diffeomorphisms. Quantitative descriptions by means of average escape times from sets as functions of the parameter are provided. Further quantitative properties are described through the speed of decay of correlations as function of the parameter.

Random endomorphisms are studied in one dimension; we show that stable one dimensional random endomorphisms occur open and dense and that in one parameter families bifurcations are typically isolated. We classify codimension one bifurcations for one dimensional random endomorphisms; we distinguish three possible kinds, the random saddle node, the random homoclinic and the random boundary bifurcation.