Strange attractors in coupled systems

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Abstract

Following our results in [1] we provide an analytical proof of the existence of strange attractors in a 4-parameter family of 4-dimensional vector fields consisting of two brusselators (a very well-known model for a chemical reaction) linearly coupled by diffusion. It is shown that such a family is a generic unfolding of a 4-dimensional nilpotent singularity of codimension 4. On the other hand, we prove that in any generic unfolding X_{μ} of an *n*-dimensional nilpotent singularity of codimension *n* there are bifurcation curves of (n-1)dimensional nilpotent singularities of codimension n-1 which are in turn generically unfolded by X_{μ} . Arguments conclude recalling (see [2] and [3]) that any generic unfolding of the 3-dimensional nilpotent singularity of codimension 3 exhibits strange attractors since it includes generic unfoldings of Shil'nikov homoclinic orbits. Our theoretical results will be compared with previous numerical evidences. Discussions will be also included about the use of similar techniques in more general contexts: coupling of more than two oscillators or the case of realistic models but different from the brusselator.

References

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