Mechanical systems with polynomial potentials of degree 4

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Abstract

We study mechanical systems defined by homogeneous polynomial potentials of degree 4 on the plane, when the potential has a definite or semi–definite sign and the energy is non–negative. We get a global description of the flow for the non–negative potential case. Some partial results are obtained for the more complicated case of non–positive potentials. In contrast with the non-negative case, we prove that the flow is complete and we find special periodic solutions, whose stability is analyzed. By using results of Morales-Ruiz we check the non–integrability of the Hamiltonian systems in terms of the potential parameters.

Homogeneous potentials appear also in the modelling of natural phenomena or processes. Along this line we may mention Contopoulos [1] and references therein who has considered third or fourth degree homogeneous perturbations of homogeneous quadratic polynomials as models in the dynamics of galaxies.

References

[1] Contopulos, G. Order and chaos in Dynamical Astronomy, *Astronomy and Astrophysics Library, Springer, Berlin* (2002).