Splitting of separatrices in pertubations of the McMillan map

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Abstract

In [1], Delshams and Ramírez-Ros considered regular perturbations of the McMillan map, a family of integrable area preserving analytic maps. It is situation with two parameters, ε, the regular one, which controls the size of the perturbations, and h, which is the Lyapunov exponent of a fixed point. They proved that the separatrices of the McMillan map split under perturbation, and gave an asymptotic formula for the area of the lobes between the invariant manifolds, provided that the parameter ε is small with respect to h. The formula was obtained applying a suitable Melnikov method.

In the work we present here, we find an asymptotic formula of the splitting of the separatrices, when ε and h are small, but independent. This formula agrees with the one in [1] when $\varepsilon \to 0$. The proof is based in matching *outer* and *inner* approximations of the manifolds, and a study of the invariant manifolds close to their singularities by means of resurgence. This allows us to explain the appearence of the Borel transform of the perturbation in the leading coefficient of the asymptotic expansion of the area. We also remark that we do not use flow box coordinates.

References

[1] A. Delshams and R. Ramírez-Ros Exponentially small splitting of separatrices for perturbed integrable Standard-like maps. *J. Nonlinear Science.*, 8:317–352, 1998.