Non-monotonic twist equations

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Abstract

For equations with one degree of freedom which have nonlinear terms in the form of algebraic or trigonometric polynomial of degree $n > 1$, period T of motion along closed phase curves may be a non-monotonic function of energy E. In this case there are energy levels for which first derivative of function $T(E)$ is equal to zero. These levels are called degenerate. Let us call the order of the first non-zero derivative of $T(E)$ as the order of degeneracy j, $j > 1$. For maximal order of degeneracy j_{max} it is shown that $j_{max} = n-1$ for algebraic case and $j_{max} = n$, $n > 1$ for trigonometric case.

For polynomials of degree 2 to 5 the values of coefficients are presented for which $T(E)$ is a non-monotonic function and also values for which $T(E)$ has extremum of the maximal order of degeneracy.

When dynamical system of this type is under time-periodic perturbation, such extrema cause the existence of degenerate resonances. The question of the structure of degenerate resonance zones is considered theoretically using analysis of averaged systems. Numerical study of Poincaré map gives a good compliance with theoretical analysis of bifurcations. Also are found of bifurcations of vortex pairs formation, which are not observed at theoretical research.

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