HAMILTONIAN STABILITY AND SUBANALYTIC GEOMETRY

Laurent NIEDERMAN[1,2]

1 Université Paris XI — Topologie et Dynamique — UMR 8628 du CNRS — Bât. 425, 91405 ORSAY Cedex France.

2 IMCCE — Astronomie et Systèmes Dynamiques — UMR 8028 du CNRS — 77 avenue Denfert-Rochereau, 75014 PARIS France.

Abstract

In the 70's, Nekhorochev proved that for an analytic nearly integrable Hamiltonian system, the action variables of the unperturbed Hamiltonian remain nearly constant over an exponentially long time with respect to the size of the perturbation, provided that the unperturbed Hamiltonian satisfies some generic transversality condition known as *steepness*. Using theorems of real subanalytic geometry, we derive a geometric criterion for steepness : a numerical function h which is real analytic around a compact set in R^n is steep if and only if its restriction to any affine subspace of R^n admits only isolated critical points[1].

We also state a necessary condition for exponential stability, which is close to steepness.

Finally, we give methods to compute lower bounds for the steepness indices of an arbitrary steep function.

Key words : Hamiltonian systems – Stability – Subanalytic Geometry – Curve Selection Lemma – Lojasiewicz's inequalities.

References

[1] L. Niederman. Exponential stability and Subanalytic Geometry. *Annales Inst. Fourier*, 56, 2006.