Hunting for quantum points near the bottom of an unharmonical Schrödinger well

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Abstract

We consider a stationary Schrödinger equation on a plain with an analytic potential, having a nondegenerated minimum (well) at the origin. We use the semi-classical (the Plank constant $\hbar$ tending to zero) Gaussian-like asymptotics constructed under a Diophantine condition on the frequencies with an *ansatz* with Hermite polynomials for the eigenfunctions [1]. In zero approximation the eigenfunction $U_{(k,l)}$ has the form of an exponent multiplied by a product of Hermite polynomials. Hence one finds a set of zeros of the function $U^{[0]}_{(k,l)}$ as a set of intersecting lines. If one takes two terms of an asymptotic series for $U_{(k,l)}$ one can see that the lines of zeros of the eigenfunction do not intersect. They form quasi-intersections. We make some numerical calculations for some potentials and investigate the lines of the zeros of eigenfunctions concentrated near the origin. Those are the eigenfunctions for the quantum vectors (k, l) having modulus 1,2,3,... Not too large. Some lines of the zeros of these eigenfunctions are the closed curves. A domain bounded by such a curve we call a quantum point.

References

[1] T.F.Pankratova Semi-classical eigenstates at the bottom of a multidimencional well. *Ann. Inst. Henri Poincaré, Physique théorique*, Vol. 62, n^o4, 1995, p.361-382.