On periodic orbits of billiards inside perturbed circular tables

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Abstract

We study two problems about the periodic orbits of the area-preserving twist billiard maps associated to monomial perturbations of circular tables of the form $x^2 + y^2 + \epsilon y^n = 1$ for some integer $n \geq 3$.

Here, $\epsilon \geq 0$ is the perturbative parameter. In the unperturbed case, the billiard map is integrable and its phase space is foliated by invariant curves. The invariant curves whose rotation number is rational: $p/q \in \mathbf{Q}$, are called p/q-resonant and do not persist under generic perturbations.

Firstly, using a standard Melnikov argument, we find the sets $\mathcal{Q}_n \subset \mathbf{N}$ such that the p/q-resonant curve breaks up at first order in ϵ if and only if $q \in \mathcal{Q}_n$. In fact, $\mathcal{Q}_{2l} = \{2, 4, \ldots, 2l\} \cup \{2, 3, \ldots, l\}$ and $\mathcal{Q}_{2l+1} = \{3, 5, \ldots, 2l+1\}$. The other resonant curves (probably) also break up, but with a much smaller amplitude. This result is contained in [1].

Secondly, we present a numerical study of some asymptotic properties of the length spectrum of the perturbed circles. We conjecture that for any odd integer $q = 2k + 1$ there exist a couple of symmetric $1/q$-periodic billiard trajectories such that their lengths are exponentially close in q. Concretely, there exists some constants $c_n > 0$ and $A_n \neq 0$ such that the difference of their lengths has the following asymptotic behaviour: $A_n \epsilon^k \exp(-c_n/k)$ when $k \nearrow \infty$ and $\epsilon \searrow 0$. We hope to relate the constants c_n to the width of the analyticity strips of some suitable functions. This is a work in progress.

References

[1] R. Ramírez-Ros. Break-up of resonant invariant curves in billiard and dual billiards associated to perturbed circular tables. *Phys. D*, 214(1):78–87, 2006.