Ergodicity and convergence to a Brownian motion in examples of Arnold diffusion

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Abstract

In the same line of research as [2], I construct examples of near-integrable Hamiltonian systems with $N + \frac{1}{2}$ degrees of freedom ($N \geq 2$), which are as smooth as possible and as "unstable" as possible. They are generated by Hamiltonian functions of the form $\frac{1}{2}(I_1^2 + \cdots + I_N^2) + f(\theta, I, t)$ with Gevrey functions f which are 1-periodic in time and arbitrarily small. The Nekhoroshev theorem [1] forces exponential stability. Still, for these particular f, one can find probability measures in the phase space such that the first $N - 1$ action variables satisfy a functional central limit theorem: when properly rescaled, they converge in law to a Brownian motion (with an exponentially small coefficient of diffusion, as should be). These probability measures are related to the product of the $2(N-1)$-fold Lebesgue measure by a measure supported on a horseshoe on the last degree of freedom. There is a related invariant measure μ for the time-1 map, which is σ-finite but not finite. One can show that, when $N = 1$ or $N = 2$, the time-1 map is μ-ergodic. For $N \geq 3$, μ-almost orbit is biasymptotic to infinity and the system is not ergodic (but there still exist orbits which are dense in the support of μ, which is the product of the first $N - 1$ degrees of freedom by the horseshoe).

References

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[2] J.-P. Marco and D. Sauzin. Wandering domains and random walks in Gevrey near-integrable systems. *Ergodic Theory & Dynamical Systems*, 24(5):1619–1666, 2004.