Homoclinic Chaos and Physical Integrability in LRS Bianchi IX Cosmologies

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May 11, 2006

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The dynamics of Bianchi IX models with two scale factors $A(t)$ and $B(t)$ is reexamined. The matter content of the model is assumed to be comoving dust plus a positive cosmological constant. We make a complete numerical construction of topological structures present in the phase space of the model, as the periodic orbits associated to a saddle-center critical point and their stable and unstable manifolds. We exhibit the homoclinic intersections of these unstable and stable manifolds, that produce homoclinic chaos in the models, leading to escape or collapse ruled by a horseshoe structure. This is possible because $A = 0$ is not a singularity of the Hamiltonian system and the dynamics in the two regions of phase space, $A > 0$ and $A < 0$, joins smoothly in $A = 0$. However if the dynamics is restricted at $A > 0$ the essential recurrence is eliminated. In this case the boundary between initial conditions leading to collapse or escape is simply the stable manifold. Therefore, the system is physically integrable in spite of its mathematical non-integrability that is numerically evidenced by the existence of the horseshoe structure.