The Hopf-saddle-node bifurcation for fixed points of 3D-diffeomorphisms: a dynamical inventory

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Abstract

A numerical study of the Hopf-saddle-node (HSN) bifurcation for fixed points of diffeomorphisms is presented. The phase space is three-dimensional and we need three parameters to give an appropriate description. The exclusion of strong resonances implies that near the central singularity the diffeomorphisms can be approximated very well by the flow of a corresponding HSN vector field case. A model map is constructed that is on the one hand near the flow of a 'versal' HSN vector field unfolding, and, on the other hand, designed 'as generic as possible' as diffeomorphisms go. Lyapunov exponents and computation of normal behavior of invariant circles by Fourier analysis are used for a systematic exploration of bifurcation patterns and dynamical phenomena of the model map. The study is centered around a 1:5 resonance taking place on a frayed quasi-periodic bifurcation boundary in a suitable parameter plane. Several subordinate quasi-periodic bifurcations of codimension 1 and 2 of invariant circles and two-tori occur in the neighborhood, leading to the formation of strange repellors. Another phenomenon of interest are chaotic regions, as well as routes that lead to quasi-periodic Hénon-like strange attractors. In particular, a quasi-periodic period doubling route is presented.