The flow of the equal-mass spatial 3-body problem in the neighborhood of the equilateral relative equilibrium

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Abstract

From a normal form analysis near the Lagrange equilateral relative equilibrium, we deduce that, up to the action of similarities and time shifts, the only relative periodic solutions which bifurcate from this solution are the (planar) homographic family and the (spatial) P_{12} family with its twelfth-order symmetry (see [4, 2]). After reduction by the rotation symmetry of the Lagrange solution, our proof of the local existence and uniqueness of P_{12} follows that of Hill's orbits in the planar circular restricted three-body problem in [3, 1]. The existence of the actual symmetric family is shown, using the uniqueness in the reduced problem. We then analyse the restriction of the reduced flow to a constant energy level in a center manifold. Such a level turns out to be a three-sphere. In an annulus of section bounded by relative periodic solutions of each family, the normal resonance along the homographic family entails that the Poincaré return map is the identity on the corresponding connected component of the boundary. Using the reflexion symmetry with respect to the plane of the relative equilibrium, we prove that, close enough to the Lagrange solution, the return map is a monotone twist map.

References

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