What algebra sees from the dynamics

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Abstract

Firstly I will shortly explain "ambiguïty theory" ("théorie de l'ambiguïté" in reference to Galois last letter). Particular cases are ordinary or differential Galois theory (in the linear case or for strongly normal extensions). We can also consider the flow of a dynamical system (a groupoïd) and "look" it with "bad glasses", that is with limited tools as topology (ordinary or Zariski topology), following some ideas of (Nakai, Loray, Vessiot, Malgrange). We get ambiguïty groupoïds as Malgrange-Vessiot Galois groupoïd [1].

I will work systematically with a "complex time". I will begin with the linear case, recall briefly differential Galois theory and explain that, in the fuchsian case, differential Galois group is the Zariski closure of the dynamic. In the general case there are problems related to exponential factors and Stokes phenomena. I will explain how we can recover the linear differential Galois group from the differential Galois groupoïd of a non-linear foliation using Malgrange's principle: "Galois groupoïd= What algebra sees from the dynamics".

I will explain after what happens in the local analytic situation in the discrete case (diffeos in dimension one fixing the origin) and the continuous case (singular foliations in dimension 2), explaining the relations between transversal structures, Lie algebras, Godbillon-Vey sequences, first integrals in strongly normal extensions and Lie-Malgrange groupoïds:$\mathcal{D}$-groupoïds. (Following mainly recent works of G. Casale and B. Malgrange).

I will end with Hamiltonian systems, Morales-Ramis theory and recent extensions by Morales-Ramis-Simo, using the point of view of $\mathcal{D}$-groupoïds.

References

[1] B. Malgrange. Le groupoïde de Galois d'un feuilletage. *L'enseignement Mathématique.*, 38(2), 2001.