The coarse-grained Gibbs entropy in dynamical problems

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Abstract

Consider a dynamical system (for example, a flow) on a phase space Γ. Let μ be an invariant measure on Γ and let ν be another measure with density $\rho : \Gamma \to [0, +\infty)$: $(d\nu = \rho d\mu)$. If ν is a probability measure, we can define the Gibbs entropy $s = s(\nu) = -\int_\Gamma \rho \log \rho d\mu$.

The flow moves the measure ν: $\nu \mapsto \nu^t$, $\rho \mapsto \rho^t$. Hence we can consider the entropy $s_t = s(\nu^t)$. According to physical intuition s_t should be an increasing function. However Poincare noticed that s_t is constant. At first glance this means that the Gibbs entropy is "not physical". But a slight modification of the construction (the so-called coarse-grained entropy) corrects the situation. I plan to discuss the definition and some properties of the coarse-grained entropy as well as its relation with the ordinary Gibbs entropy.