Quantum Manifestations of Homoclinic and Heteroclinic Motions

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Abstract

The pioneering work of Poincaré showed the importance of homoclinic and heteroclinic motions to rationalize chaotic motions in Hamiltonian systems, when studied classically. However, nature is quantum at microscopic (atomic and molecular) level. The question then arises as if these motions have a similar prominent role in quantum mechanics, a theory where the dynamics are governed by a linear (Schrödinger) equation. We will conclusively show at the conference that the answer to the above question is yes.

The tool to carry out our investigation are scarred wave functions. The term "scar" was coined by Heller in 1984 (Phys. Rev. Lett. **53**, 1515) to describe an enhanced localization of quantum probability along short unstable periodic orbits (PO) on certain eigenfunctions of classically chaotic systems (whose probability was expected, loosely speaking, to be uniformly distributed in all available phase space). This constitutes the most spectacular demonstration that quantum knows about classical mechanics, when the dynamics are chaotic. Heller explained this accumulation of probability using wave packets running along the PO, as the result of constructive interference of the recurrences when the associated classical circuit along the PO was properly quantized (equal to an integer times the value of Planck's constant). This theory did not take care of the effect of the dynamics in the perpendicular direction induced by the Lyapunov exponent. Our work show, that this dynamics, responsible for the homoclinic and heteroclinic excursions related to the scarring PO also produce localization effects similar to scarring when properly quantized. This implies quantization of phase space areas defined by the different homoclinic and heteroclinic orbits.