Low Prandtl number convection in rotating spherical shells

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Abstract

In a fast rotating spherical shell, the onset of thermal convection depends strongly on the Prandtl and Taylor numbers. There are a lot of theoretical and numerical papers devoted to study the spiralling columnar modes of convection dominant at moderate and high Prandtl numbers, mainly with stress-free boundary conditions because they are relevant in astrophysical problems. Low Prandtl number problems are more difficult because they model planetary metallic cores with thin Ekman boundary-layers near the rigid boundaries. Finding a solution requires very high resolutions, and for this reason there have been very few attempts to compute the marginal modes in this range of parameters.

An accurate iterative method to find the leading spectra of the linearised equations with very small Prandtl numbers and large rotation rates is presented. The problem is solved by applying a collocation method in the radial direction, and expanding in spherical harmonic series in the two angles. Then the linearised equations split into their azimuthal Fourier coefficients. If the equations for a given azimuthal mode m are written as $\dot{\mathbf{x}}_m = \mathcal{A}_m \mathbf{x}_m$, where $\mathbf{x}_m$ are all the amplitudes of the variables in spherical harmonics of order m, the rightmost eigenvalues of $\mathcal{A}_m$ are computed either by exponentiation by time evolution or by a more efficient complex shift-invert transform.

It is found that at moderate Taylor numbers Ta, the dominant patterns of convection are the well known drifting equatorially-attached modes, but they are superseded by multi-cellular spiralling outer-wall-attached modes as the rotation rate is increased. In consequence, the convection spreads to high latitudes affecting the body of the fluid, and the critical Rayleigh number R_c fulfils a power-law dependence $R_c \sim Ta^{\alpha}$, as for moderate Prandtl number fluids.