Straightening the Flow in a Hamilton-Jacobi Equation

Carme Olivé[1] Tere M. Seara[2]

1 Departament d'Enginyeria Informàtica i Matemàtiques. Universitat Rovira i Virgili. Avda Països Catalans 26, 43007 Tarragona. carme.olive@urv.net

2 Departament de Matemàtica Aplicada I. Universitat Politècnica de Catalunya. Av. Diagonal 647, 08028, Barcelona. tere.m-seara@upc.edu

Abstract

The two-dimensional stable and unstable manifolds of the hyperbolic $2\pi\varepsilon$-periodic solution in the rapidly forced pendulum with Hamiltonian $H_{\mu,\varepsilon}(q,p,t)$ can be considered as graphs of differentials $p = \partial_q S^{\pm}(q,t,\mu,\varepsilon)$, where S^+ and S^- are $2\pi\varepsilon$-periodic in t and analytic for q in certain domains, and satisfy the Hamilton-Jacobi equation $\partial_t S + H_{\mu,\varepsilon}(q, \partial_q S, t) = 0$ with asymptotic conditions $\lim_{q\to 0,2\pi} \partial_q S^{\mp}(q,t,\mu,\varepsilon) = 0$ (see for example [1]).

These functions are approximated for different expressions depending on the complex domain of q, and the difference between them is studied via the Resurgence Theory (see [2]). From all of this, and using complex matching techniques, we will obtain a change of variables which congugates the vector field $\partial_t + \frac{1}{2}\left(\partial_q S^+ + \partial_q S^-\right)\partial_q$ and the straightened $\partial_t + \partial_q$. This result will allow us to obtain an exponentially small formula as $\varepsilon \to 0$ for the splitting of invariant manifolds.

References

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