On Birkhoff Theorems for Lagrangian invariant tori with closed orbits

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Abstract

We study Birkhoff Theorems for Lagrangian tori with closed orbits which are invariant by the Hamiltonian flow of convex, superlinear Hamiltonians in T^n. The first Birkhoff Theorem (that is in fact a conjecture) says that any continuous, minimizing, Lagrangian, invariant torus of a convex, superlinear Hamiltonian is a graph of the canonical projection. This theorem was proved by Bialy and Polterovich in 1987 for invariant tori of symmetric Finsler metrics in T^2.

The second Birkhoff Theorem (again, still a conjecture) states that smooth, Lagrangian invariant tori which are homologous to the zero section are graphs of the canonical projection. This assertion was proved by Bialy in 1989 for continuous, Lagrangian, two dimensional tori of symmetric Finsler metrics in T^2, provided that the Lagrangian tori have no closed orbits. Later in 1990-92, Bialy-Polterovich proved the second Birkhoff Theorem for n-dimensional tori provided that the Lagrangian torus is C^3 and that every orbit in the torus is dense.

We exhibit a counterexample to the first Birkhoff Theorem for continuous, two dimensional, Lagrangian invariant tori with closed orbits. However, we show that under certain hypothesis the first Theorem holds for continuous invariant tori of Finsler metrics in T^2. We also show that the second Birkhoff Theorem for C^1, invariant tori with closed orbits is true for symmetric Finsler metrics in T^2.

References

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