Curvatures and focal points of dynamical Lagrangian distributions and their reduction by first integrals.

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Abstract

We consider the basic differential invariants of the pair (Hamiltonian system, Lagrangian distribution) w.r.t. the action of the group of symplectomorphisms of the ambient symplectic manifold. These invariants: the *generalized curvature operator* and the corresponding *curvature form*, generalize the notion of the classical curvature tensor and the Ricci curvature in Riemannian geometry. In particular, they can be used for the localization of the focal points along extremals of the corresponding variational problems.

We present the results concerning the variation of this curvatures and the structure of focal points related to the reduction of the problem by arbitrary first integrals in involution, illustrating the observed phenomena by systems of Classical Mechanics and in particular by the classical N-body problem.

References

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