Oscillatory Solution to the Three-Body Problem

Mitsuru Shibayama[1]

1 Department of Mathematics, Faculty of Science, Kyoto University, Kyoto 606-8502, Japan

Abstract

Consider the planar three-body problem with equal masses:

$$\frac{d^2\mathbf{x}_k}{dt^2} = \frac{\partial U}{\partial \mathbf{x}_k}, \quad \mathbf{x}_k \in \mathbb{R}^2, k = 1, 2, 3$$

where

$$U(\mathbf{x}) = \sum_{i<j} \frac{1}{r_{ij}}, \quad r_{ij} = \|\mathbf{x}_i - \mathbf{x}_j\|.$$

A solution is called a *oscillatory solution* if it satisfies

$$\limsup_{t\to\infty} \max_{i<j} r_{ij} = \infty$$
$$\liminf_{t\to\infty} \max_{i<j} r_{ij} < \infty.$$

Oscillatory solutions were discovered by Sitonikov, Alekseev, Xia, etc.
As far as these solutions are concerned, one or two of three particles repeatly go away. We will prove the existence of a new oscillatory solution. Its behavoir is the following. At first m_1 and m_2 are near each other and m_3 approaches them, and then near triple collision occurs. After that m_1 go far away from the others and then go back, After that m_2 go further from the others, ...
This is a new oscillatory solution.