The Generalized Alignment Index (GALI) method: Detecting order and chaos in conservative dynamical systems

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Abstract

We investigate the dynamics of conservative dynamical systems by studying the evolution of volume elements formed by deviation vectors about their orbits. The behavior of these volumes is strongly influenced by the regular or chaotic nature of the motion. The different time evolution of these volumes can be used to identify rapidly and efficiently the nature of the dynamics, leading to the introduction of quantities that clearly distinguish between chaotic behavior and quasiperiodic motion. More specifically we define the Generalized Alignment Index of order k (GALI_k) as the volume of a generalized parallelepiped, whose edges are k initially linearly independent unit deviation vectors from the studied orbit. We show analytically and verify numerically on particular examples of N degree of freedom Hamiltonian systems and $2N$–dimensional symplectic mapping that, for chaotic orbits, GALI_k tends exponentially to zero with exponents that involve the values of several Lyapunov exponents, while in the case of ordered orbits, GALI_k fluctuates around non–zero values for $2 \leq k \leq N$ and goes to zero for $N < k \leq 2N$ following power laws that depend on the dimension of the torus and the number of deviation vectors initially tangent to the torus. The GALI_k is a generalization of the Smaller Alignment Index (SALI) as $\mathrm{GALI}_2 \propto \mathrm{SALI}$. However, GALI_k provides significantly more detailed information on the local dynamics of the system, allows for a faster and clearer distinction between order and chaos than SALI and works even in cases where the SALI method faced difficulties.