A Lie-Algebra version
of Classical or Quantum Hamiltonian
Perturbation Theory & Control

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Abstract

We consider a Hamiltonian which leaves invariant a sub-Lie algebra $\mathbb{V}_0$ of the Lie algebra of "observables" $\mathbb{V}$, which happens for instance when the Hamiltonian is "locally integrable". For any perturbation of this Hamiltonian, we give an expression for the sub-Lie algebra isomorphic to $\mathbb{V}_0$ which is invariant by the perturbed system. A simpler problem is to "slightly" modify the perturbed system (by an additive "control" term, for instance quadratic in the size of the perturbation) such that the above isomorphism is simple to compute. This theory generalizes a recent control of Hamiltonian systems that has been already applied in some physical examples. Here we give some additional examples, in Classical or Quantum mechanics.