BDSM15, December 16th, 2015

Facultat de Matemàtiques, UB

Room T1

Abstracts

Carles Simó (Universitat de Barcelona): The invariant manifolds at infinity of the RTBP and the boundaries of bounded motion. The invariant manifolds of the periodic orbit at infinity in the planar circular RTBP are studied. To this end we consider the intersection of the manifolds with the passage through the barycentric pericentre. The intersections of the stable and unstable manifolds have a common even part, which can be seen as a displaced version of the two-body problem, and an odd part which gives rise to a splitting. The theoretical formulas, obtained for Jacobi constant C large enough, are compared to direct numerical computations, showing improved agreement when C increases. A return map to the pericentre passage is derived and using an approximation by standard-like maps one can make a prediction of the location of the boundaries of bounded motion. This result is compared to numerical estimates, again improving for C increasing. Several anomalous phenomena are described.

Rafael de la Llave (GeorgiaTech): Quasi-periodic solutions for state dependent delay equations. We consider delay differential equations in which the delay depends on the state of the system These equations appear naturally in electrodynamics for particles interacting with retarded potentials (the delay is proportional to the distance) as well as in several biological models. We note that for these equations the phase space is infinite dimensional and not easy to describe. Questions such existence, uniqueness, dependendence on parameters are still puzzling.
We develop a theory of quasi-periodic solutions that bypasses the questions of existence for general initial data. We develop a functional equation for the quasiperiodic equations and study them by functional analysis methods. The main results are stated in an a-posteriori format that states that given pproximate solutions that satisfy some explicit non-degeneneracy solutions, there are true solutions nearby. This can be used to justify some numerical solutions that have been produced. We show that in a one-parameter family, there are smooth solutuions. Furthermore, we can find a large measure set where the quasi-peridodic solution are analytic. We conjecture that this is regularity is optimal. We also develop a theory of stable/unstable manifolds. This is joint work with Xiaolong He.

Patrick Bonckaert (University of Hasselt): Resonant planar saddle points. After recalling some standard facts about a $p : -q$ resonant saddle singular point of a smooth planar vector field, we provide approximations of the normal form in the analytic category. We discuss what happens when tightening the approximation. We encounter a fixed point problem in some Banach space of analytic functions, which we can solve in a thrifty way by studying the degenerate fixed point $(0, 0)$ of the planar diffeomorphism $G(x, y) = (x + xy, y + x + xy)$, more particularly its invariant manifold (graph) $x = h(y)$.
Next we consider the linearization of the resonant normal form using degenerate variables such as, for example, $(\log |x|).x$. We obtain a linearizing transformation that is of a certain Gevrey-class in those variables.
Finally, we would like to understand what happens when unfolding the resonance by a small parameter ε, for example if the eigenvalues are 1 and $-1 + \varepsilon$, and we end up with some unsolved questions.

Vassili Gelfreich (University of Warwick): Stokes phenomenon, singularly perturbed differential equations and bifurcations. This talk is about relations between geometric singular perturbation theory, bifurcation theory and Stokes phenomenon. We discuss some recent

developments in the theory, an application to the Hamiltonian-Hopf bifurcation and illustrate the general theory with examples.
It is well known that bifurcation problems can often be studied with the help of singular perturbation theory. In the singular perturbation theory, slow manifolds are at the centre of the interest. In the Hamiltonian case the slow manifold can be normally elliptic and therefore is not stable under a small perturbation. The disappearance of the invariant manifolds is accompanied by the disappearance of homoclinic orbits. In the analytic theory, the separatrix splitting is exponentially small with respect to the perturbation parameter. The study of the separatrix splitting naturally leads to the Stokes phenomena where the asymptotic behaviour of analytic functions differ in different regions of the complex plane. The talk is based on joint works with L.Lerman, and R. Barrio, A.Champneys and T.Lazaro.

Xavier Cabré (ICREA and Universitat Politècnica de Catalunya): Curves and surfaces with constant nonlocal mean curvature. We are concerned with hypersurfaces of $\mathbb{R}^N$ with constant nonlocal (or fractional) mean curvature. This is the equation associated to critical points of the fractional perimeter under a volume constraint. Our results are twofold. First we prove the nonlocal analogue of the Alexandrov result characterizing spheres as the only closed embedded hypersurfaces in $\mathbb{R}^N$ with constant mean curvature. Here we use the moving planes method. Our second result establishes the existence of periodic bands or "cylinders" in $\mathbb{R}^2$ with constant nonlocal mean curvature and bifurcating from a straight band. These are Delaunay type bands in the nonlocal setting. Here we use a Lyapunov-Schmidt procedure for a quasilinear type fractional elliptic equation.
(This is joint work with Mouhamed M. Fall, Joan Sol-Morales, and Tobias Weth)

Yannick Sire (Johns Hopkins University): KAM theory for ill-posed PDEs. I will develop a KAM theory for whiskered tori, which applies even to ill-posed PDEs. The KAM theory is based on automatic reducibility on the center subspace and some functional analytic properties of the hyperbolic bundles. It does not use action-angle variables and is written in an a posteriori format. I will apply it to equations coming from fluid dynamics. This is joint work with Rafael de la Llave.

Amadeu Delshams (Universitat Politècnica de Catalunya): Global instability in the periodic cubic defocusing NLS equation using non-transverse heteroclinic chains. We introduce a new geometric mechanism to explain the global instability in the periodic cubic defocusing NLS equation found by Colliander-Keel-Staffilani-Takaoka-Tao (2010) and also developed by Guardia-Kaloshin (2015). This mechanism relies on the shadowing of a sequence of invariant tori connected along non-transverse heteroclinic orbits, which satisfy some geometric restrictions, and can be readily applied to other dynamical systems, particularly infinite-dimensional Hamiltonian systems. This is a joint work with A. Simon and P. Zgliczynski.