# Reconfiguration of spacecraft formations in the vicinity of Libration points.

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# Outline

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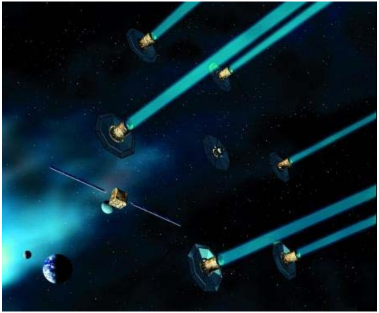
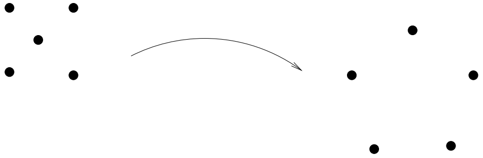
# Formations of spacecraft



1. A set of $N$ spacecraft flying in harmony
2. A virtual larger instrument

# General Purpose

Reconfiguration and deployment of spacecraft formations



Examples:

- Reorientation of formations
- Swap spacecraft
- Rendevous of stacks

# The reconfiguration problem

## Problem to solve

- Reconfiguration of $N$ spacecraft in a fixed time $T$ (fixed initial and final position)
- Minimizing fuel consumption
- Collision avoidance
- Assume the spacecraft are in a Halo orbit about $L_2$ (120,000 km of $z$-amplitude in the Sun-Earth system). **The procedure is similar if they are in another libration point orbit, about the Earth or in free space.**

# Outline

# Governing equations

We consider the linearized equations about the nonlinear orbit, due to the size of the formation:

$$\dot{\mathbf{X}}_i(t) = A(t)\mathbf{X}_i(t)$$

Add of a control $U_i(t) = (0, 0, 0, u_i^x(t), u_i^y(t), u_i^z(t))^t$

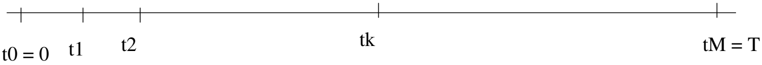
$$\dot{\mathbf{X}}_i(t) = A(t)\mathbf{X}_i(t) + U_i(t)$$

$$\mathbf{X}_i(0) = \mathbf{X}_i^0, \quad \mathbf{X}_i(T) = \mathbf{X}_i^T$$

**Objective**: find the controls $u_i$, $i = 1 \dots N$.

# The finite element method

- The FEM is usually a discretization on space, but we considere it on time: we divide the time interval $[0, T]$ in a number $M$ of smaller intervals (elements) (can be different for each spacecraft).




- A delta-v applied to each node.
- Formulate the optimal problem. For each spacecraft, we considere the functional to be minimized:

$$J = \sum_{i=0}^{N} J_i, \quad J_i = \sum_{k=0}^{M} ||\Delta v_{i,k}||,$$

where $|| * ||$ denote the Euclidean norm.

# Comments on the functional to be minimized

Derivatives of

$$J_i = \sum_{k=0}^{M} \rho_{i,k} ||\Delta v_{i,k}||$$

are ill conditioned when $||\Delta v||$ are small.

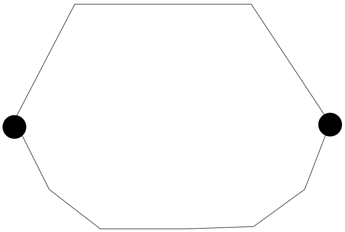
Alternative function to minimize:

$$J_i = \sum_{k=0}^{M} \rho_{i,k} ||\Delta v_{i,k}||^2.$$

- Derivatives can be calculated in any case.
- The value of $J_i$ is directly related with fuel consume.

# Obtention of the elements

- Discretization on time:
  - Divide the time interval $[0, T]$ in a number $M$ of smaller intervals (elements)
  - Possibility of use of elements of different length in the same discretization
  - The discretization can be different for each spacecraft.
- A delta-v applied to each node.



# Collision avoidance

- Procedure:
  - A sphere evolves each spacecraft.
  - The spheres cannot intersect at any time.
- Implementation:
  - Test of the restriction on each element, for each pair of spacecraft.
  - Restrictions enter in the optimal problem in a natural way.
  - Possibility of adding other restrictions in a similar way.

# The uncoupled system

Objective: uncouple the system

$$\dot{\mathbf{X}}(t) = A(t)\mathbf{X}(t). \quad (1)$$

Jordan form of $A(t)$:

$$D(t) = \begin{pmatrix} \lambda_1(t) & & & & & \\ & -\lambda_1(t) & & & & \\ & & 0 & \lambda_2(t) & & \\ & & -\lambda_2(t) & 0 & & \\ & & & & 0 & \lambda_3(t) \\ & & & & -\lambda_3(t) & 0 \end{pmatrix}. \quad (2)$$

$$\dot{\mathbf{Z}} = D(t)\mathbf{Z}.$$

# Equations for the uncoupled system

$$\dot{z}_1 = \lambda_1(t) z_1, \qquad \dot{z}_2 = -\lambda_1(t) z_2 \tag{3}$$

$$\ddot{z}_1 = (\dot{\lambda}_1(t) + \lambda_1^2(t)) z_1, \qquad \ddot{z}_2 = (-\dot{\lambda}_1(t) + \lambda_1^2(t)) z_2. \tag{4}$$

$$\dot{z}_3 = \lambda_2(t) z_4, \qquad \dot{z}_4 = -\lambda_2(t) z_3, \tag{5}$$

$$\ddot{z}_3 = \frac{\dot{\lambda}_2}{\lambda_2} \dot{z}_3 - \lambda_2^2 z_3, \quad \ddot{z}_4 = \frac{\dot{\lambda}_2}{\lambda_2} \dot{z}_4 - \lambda_2^2 z_4. \tag{6}$$

# Controls for uncoupled coordinates

$$\dot{z}_1 = \lambda_1(t) z_1 + U_1, \tag{7}$$

$$\ddot{z}_1 = (\dot{\lambda}_1(t) + \lambda_1^2(t)) z_1 + \lambda_1(t) U_1 + \dot{U}_1.$$

$$U_1(u_1) = U_0 + (-\frac{\lambda_1}{\dot{u}_1} U_0 + \frac{1}{\dot{u}_1} u_0)(u_1 - u_0), + O((u_1 - u_0)^2),$$

# Obtain the change of coordinates

Change of coordinates: $\mathbf{x} = P(t)\mathbf{z}$
$P(t)$ is solution of

$$\dot{P}(t) = A(t)P(t) - P(t)D(t). \tag{8}$$

Solved numerically using that:

$$P^{-1}(0)A(0)P(0) = D(0).$$

$P(0)$ constructed using the eigenvectors of $A(0)$.

# The optimal problem

## Functional to be minimized

$$J = \sum_{i=0}^{N} J_i, \quad J_i = \sum_{k=0}^{M} ||\Delta v_{i,k}||$$

## Constraints

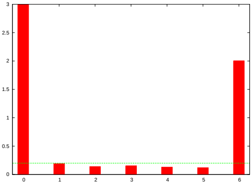
- Collision avoidance
- Geometrical constraints

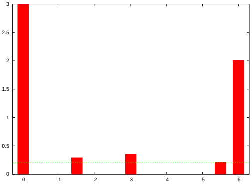
# Outline

# Changing the mesh discretization

## (I) Taking out nodes

- Remove some of the nodes which have a delta-v less than a tolerance $T_m$.
  - If there is only an isolated delta-v less than $T_m$, take off the node.
  - If there are consecutive delta-v less than $T_m$, decrease the density of nodes.

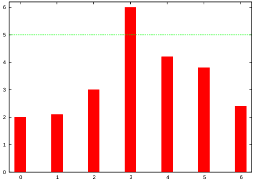


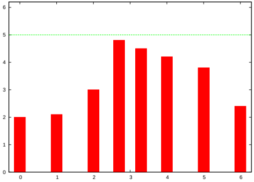



# Changing the mesh discretization

## (II) Adding nodes

- Add nodes when there is a delta-v greater than $T_M$.

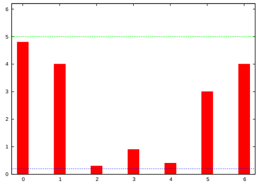


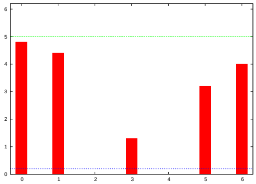



# Changing the mesh discretization

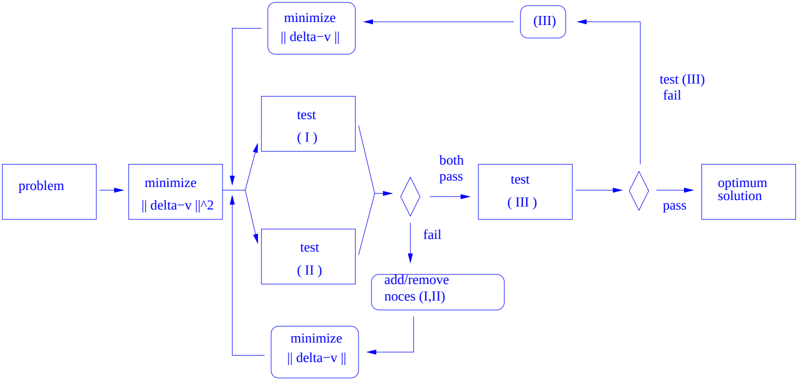
## (III) Remeshing depending on relative values of delta-v

- This procedure is applied when there are no nodes with a delta-v smaller than $T_m$ or higher than $T_M$.
- Remove delta-v when they are very small when compared with the ones in a neighborhood.
- This is useful in situations where the bang-bang control is the optimal solution.



# Flow chart of the algorithm



# Bang-bang versus low thrust

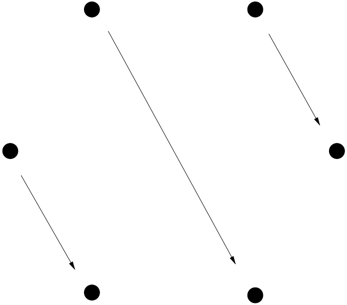
- **Bang-bang:**
  - When there is no collision risk with rectilinear trajectories, the optimum trajectory is a bang-bang control for each spacecraft.
  - The bang-bang control is the critical case, because almost all delta-v are zero.
  - After a few iterations, all the nodes are removed.
- **Low thrust:**
  - Adding nodes, the total amount of delta-v is reduced and the trajectory tends to low thrust when adding more nodes.

# EXAMPLES

1. Parallel shift ending with a bang-bang solution
2. Switching satellites in the TPF formation using low thrust
3. Test bench stress testing example
4. Rendevous of two stacks

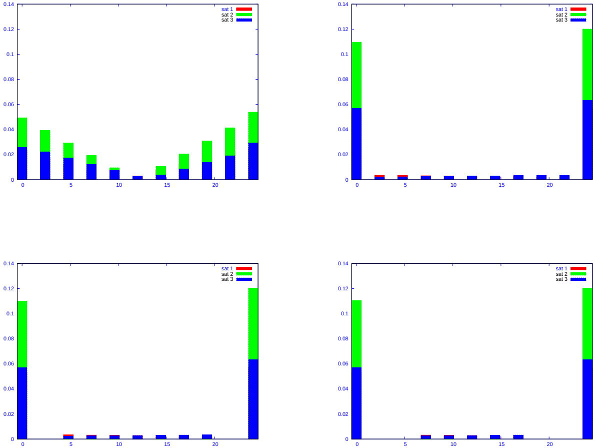
# Parallel shift example

- First configuration: 3 spacecraft in the vertices of a hexagon
- Last configuration: the spacecraft in the other vertices of the hexagon



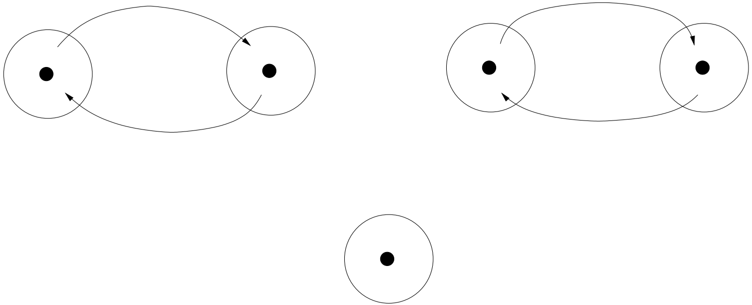
# Parallel shift example

Delta-v in the successive iterations:



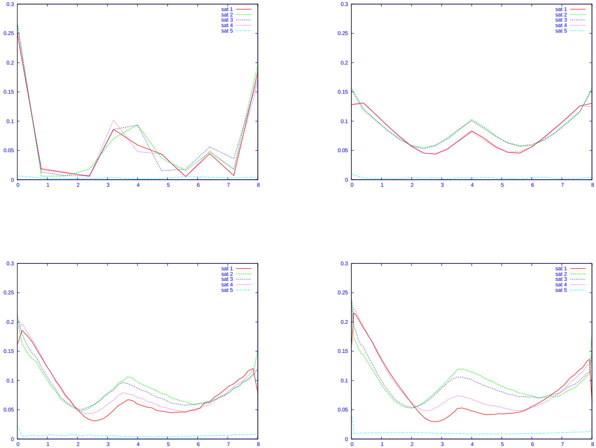
# A low thrust example of reconfiguration

- Example using the Terrestrial Planet Finder configuration
- Collision of spacecraft with a naive bang-bang control
- Switch of the spacecraft in the baseline in pairs



# A low thrust example of reconfiguration

Delta-v obtained with different number of elements



# Test bench example

- Example with 12 spacecraft, in the vertices of two regular perpendicular hexagons.
- Change the position of the spacecraft, rotating them 120 degree (multiple collision with rectilinear trajectories).

# Test bench example

Costs:

- With a security distance of 10 meters, there is no collision with rectilinear trajectories, and the solution is low thrust.
- With more than 10 meters of security distance, the cost increases with the security distance.

| Sec. distance | 10 elem | 20 elem | 50 elem | 100 elem |
|---|---|---|---|---|
| 10 | 0.24 | 0.24 | 0.24 | 0.24 |
| 20 | 1.66 | 1.43 | 1.34 | 1.29 |
| 30 | 1.92 | 1.64 | 1.45 | 1.41 |
| 40 | 2.01 | 1.89 | 1.76 | 1.73 |

# Rendevous of two stacks

- Rendevous of two stacks of spacecraft separated by 1000 meters.
- One stack with 3 spacecraft and another stack with 4 spacecraft.
- The security distance increases from 0 to the final security distance.
- The optimal is a bang-bang trajectory for each spacecraft.

# Outline

# Adaptive re-meshing advantages

- Control of the error
- Efficient computation
- Capability to add more spacecraft

# Schema of re-meshing

Initial mesh

# Schema of re-meshing

Initial mesh

↓

Solution (FEM)

# Schema of re-meshing

Initial mesh

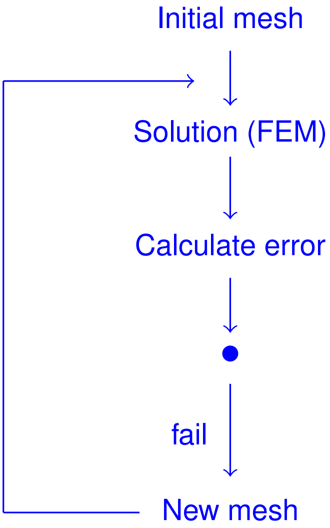
↓

Solution (FEM)

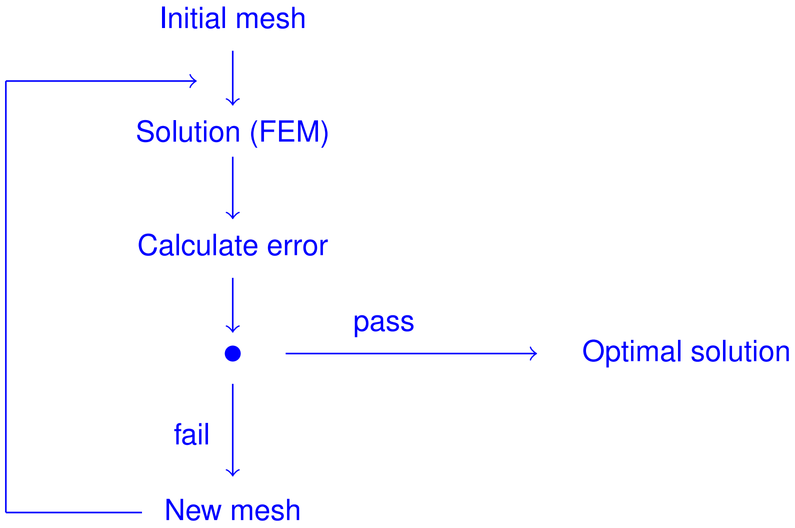
↓

Calculate error

# Schema of re-meshing



# Schema of re-meshing



# Estimation of the error

## The two gradients

- Solution of the finite element method (approximated gradient)
  - Constant on each element (linear elements)
- Obtained by integration of the equations (exact gradient)
  - $\dot{\mathbf{x}} = A(t)\mathbf{x}$

## Comparison of the gradients

On each element, the error is calculated:

$$error_k = ||\nabla_1 - \nabla_2||_{L_2} = \left(\int_{t_k}^{t_{k+1}} (v_1 - v_2) \cdot (v_1 - v_2) dt\right)^{1/2}.$$

## Acceptability criteria

Compare the total error of the mesh ($||e||$) with the maximum length of the elements ($h$)
General criteria: acceptance if

$$||e|| \leq \alpha h.$$

## Obtention of the new mesh

Li and Bettess strategy:

- Number of elements of the new discretization:

$$\hat{M} = (\nu||u||)^{-1}(\sum_{k=1}^{M} ||e||_k^{2/3})^{3/2}$$

- New length for the elements:

$$\hat{h}_k = (\frac{\nu||u||}{\hat{M}||e||_k})^{2/3} h_k.$$

**EXAMPLES**

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2. Switching satellites in the TPF formation using low thrust

# Parallel shift example

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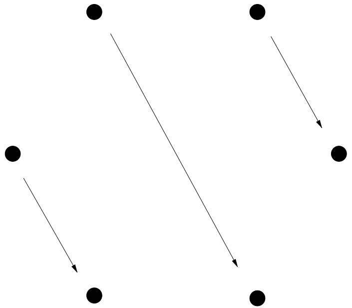
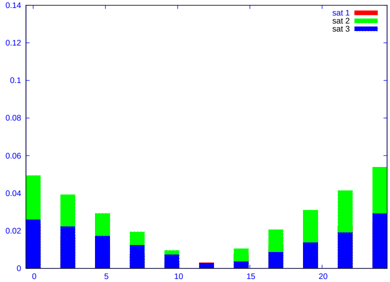


# Parallel shift example

Delta-v in the successive iterations:



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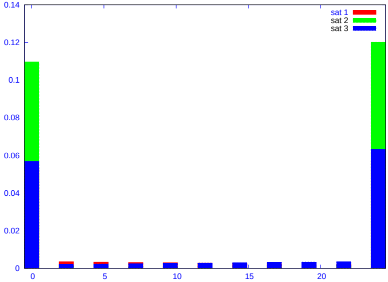

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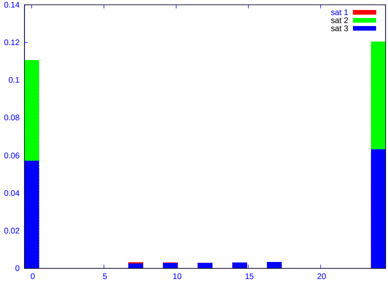

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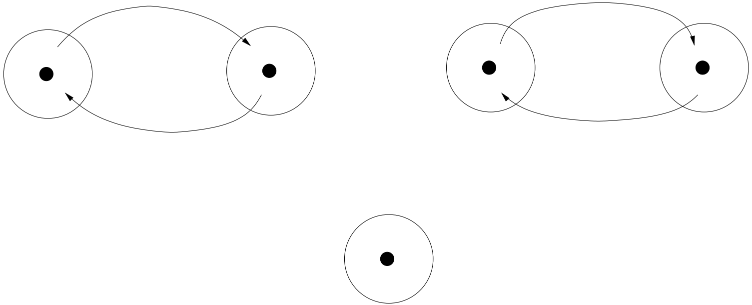
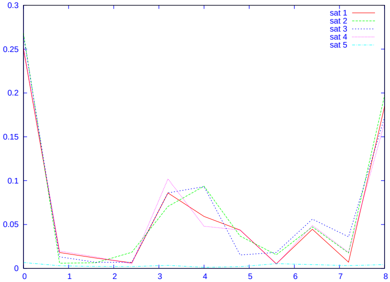


# Switching satellites

## Delta-v obtained in successive iterations



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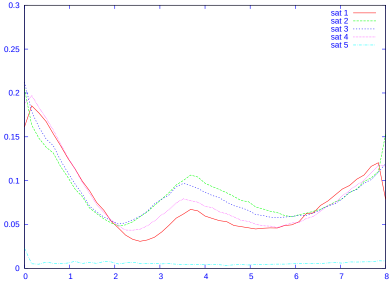

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Delta-v obtained in successive iterations