Structure in narrow rings: The Scattering approach

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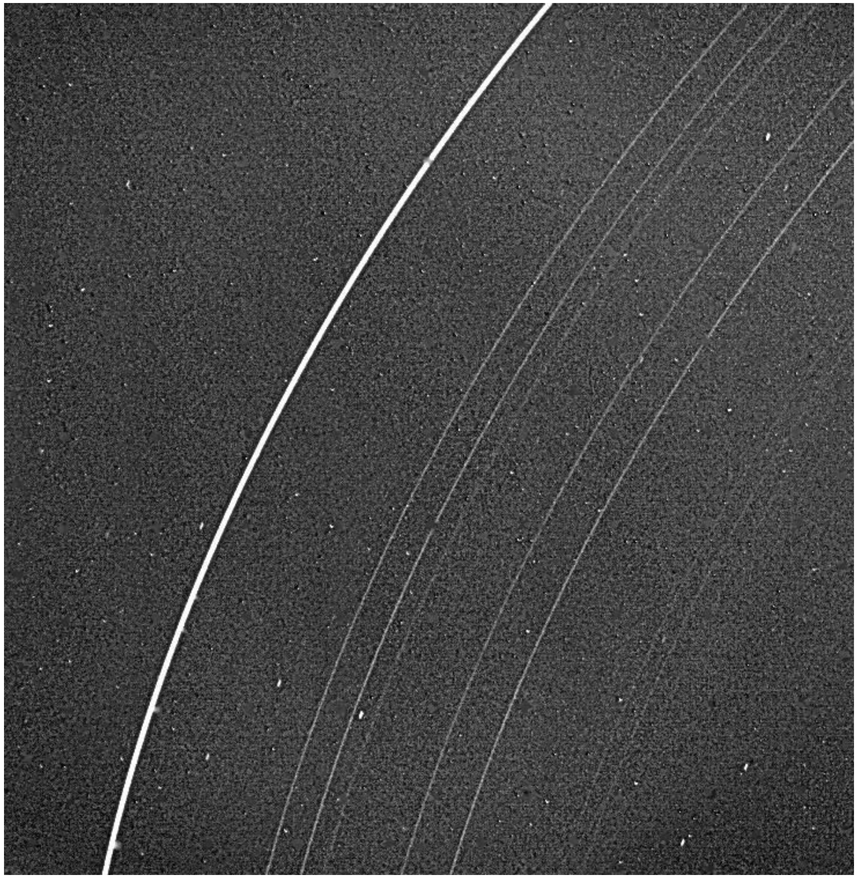
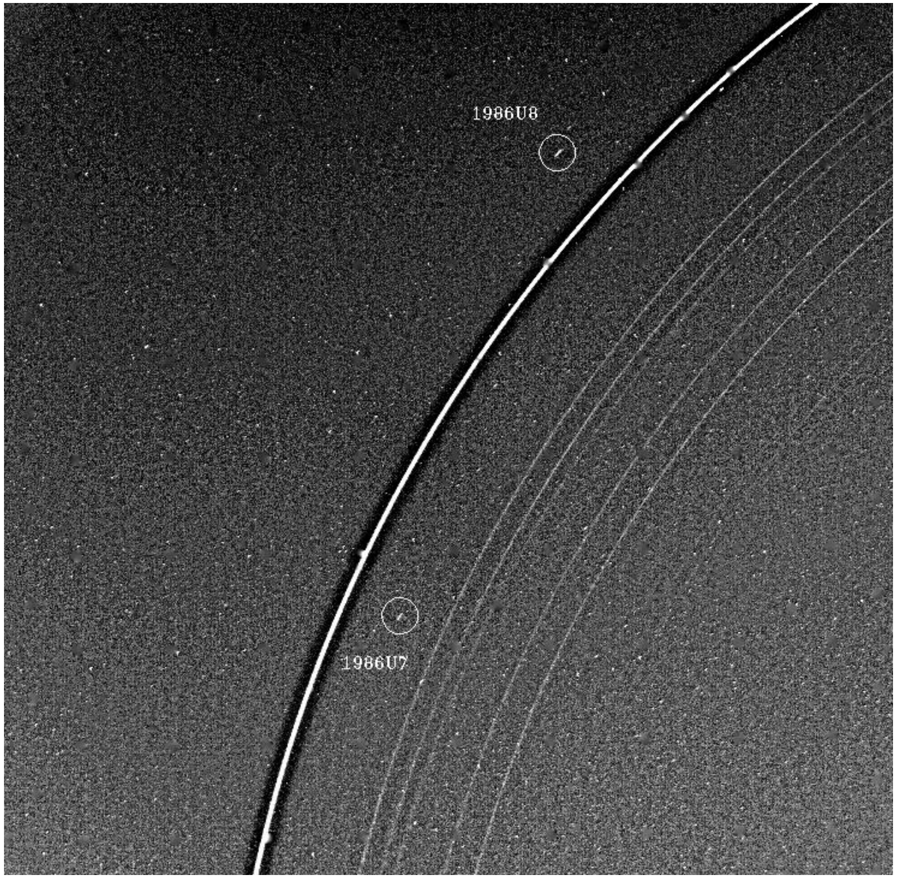
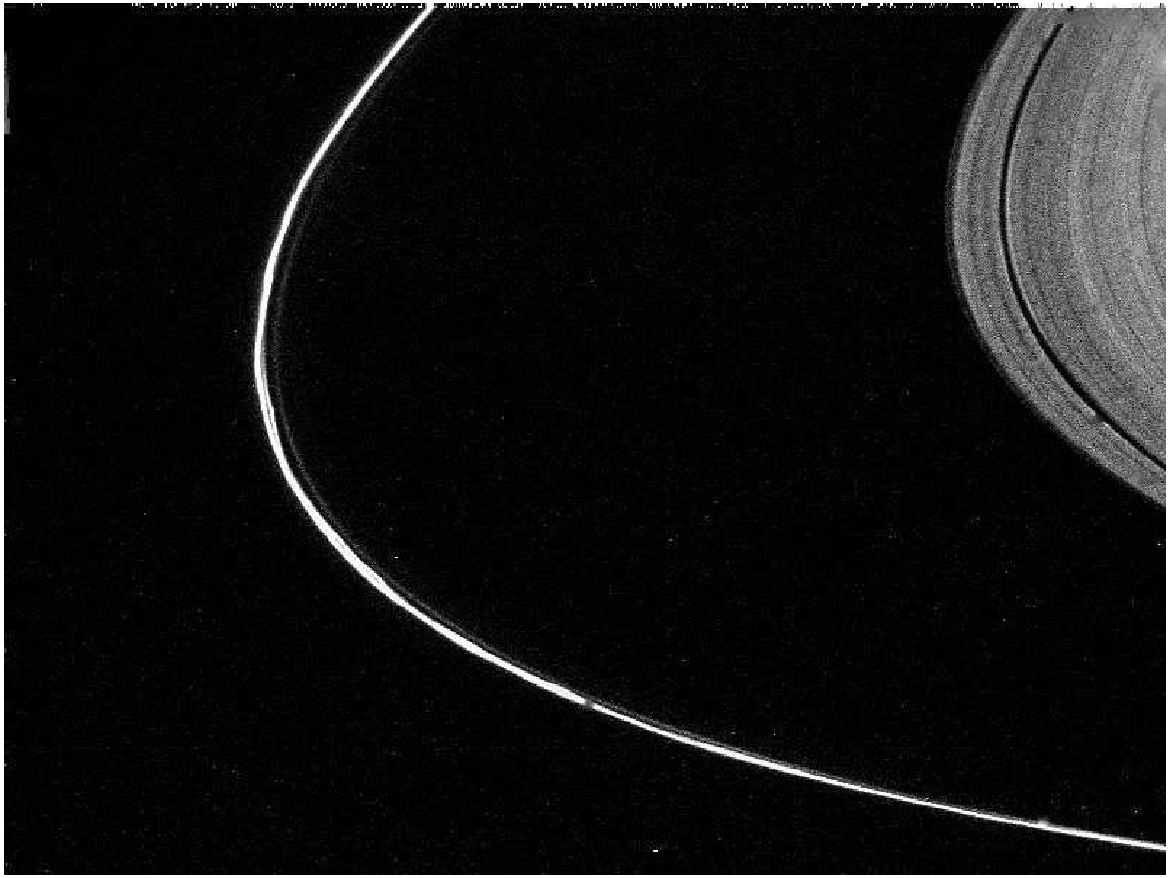


Uranus rings
PIA01977 (NASA/JPL/Space Science Institute)

Observations: narrow rings

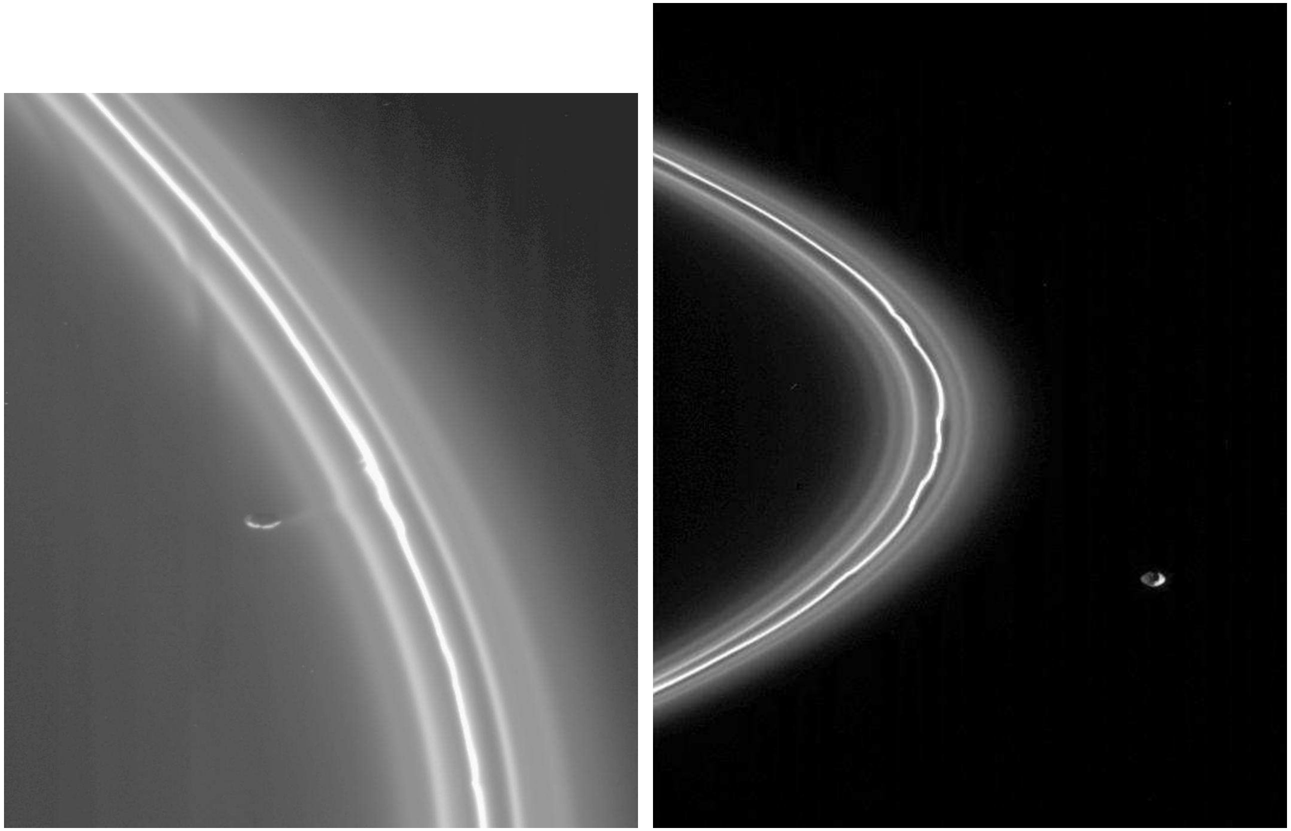


Uranus rings and shepherds
PIA01976 (NASA/JPL/Space Science Institute)



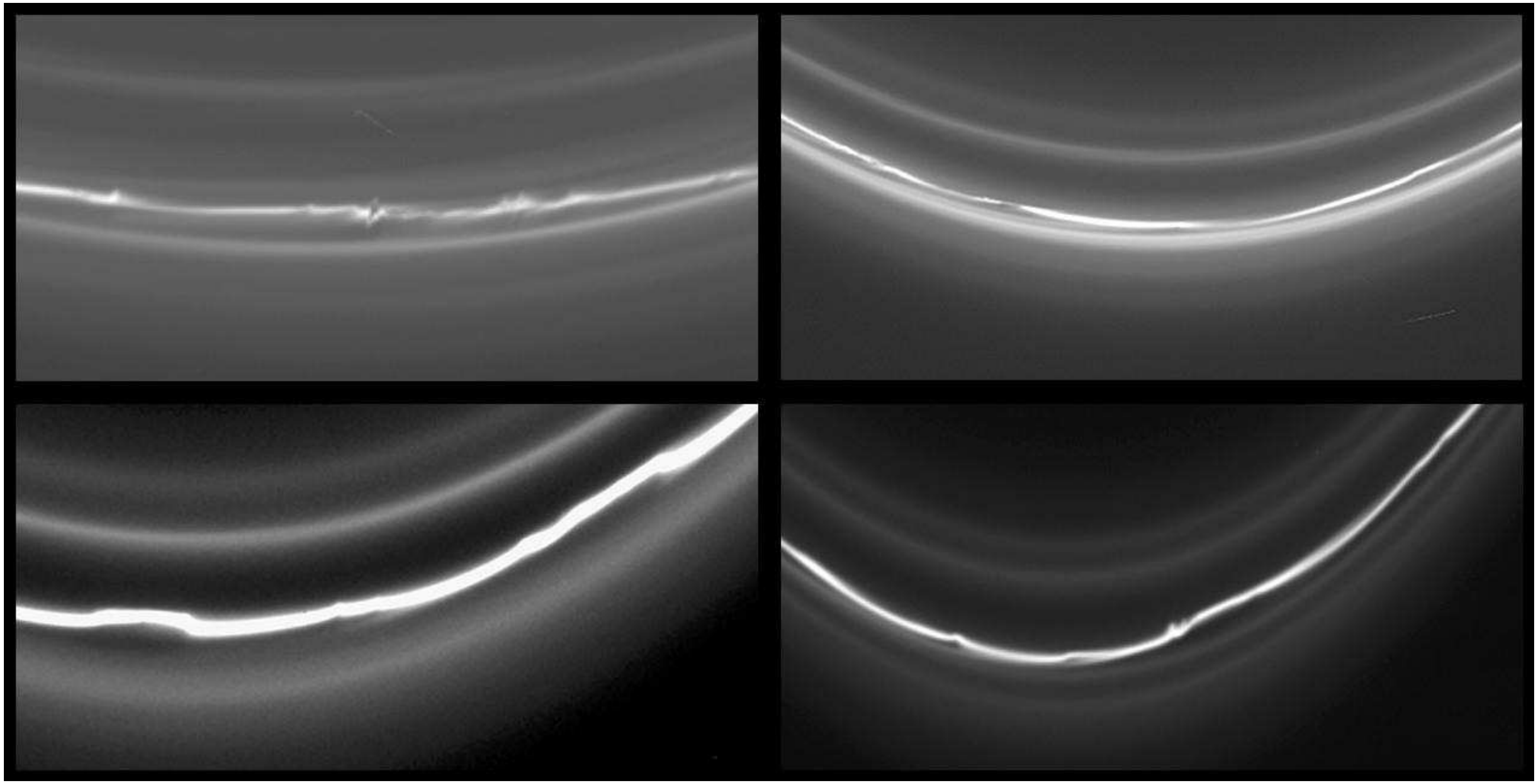
Saturn's F ring
PIA02292 (NASA/JPL/Space Science Institute)

Observations: narrow rings



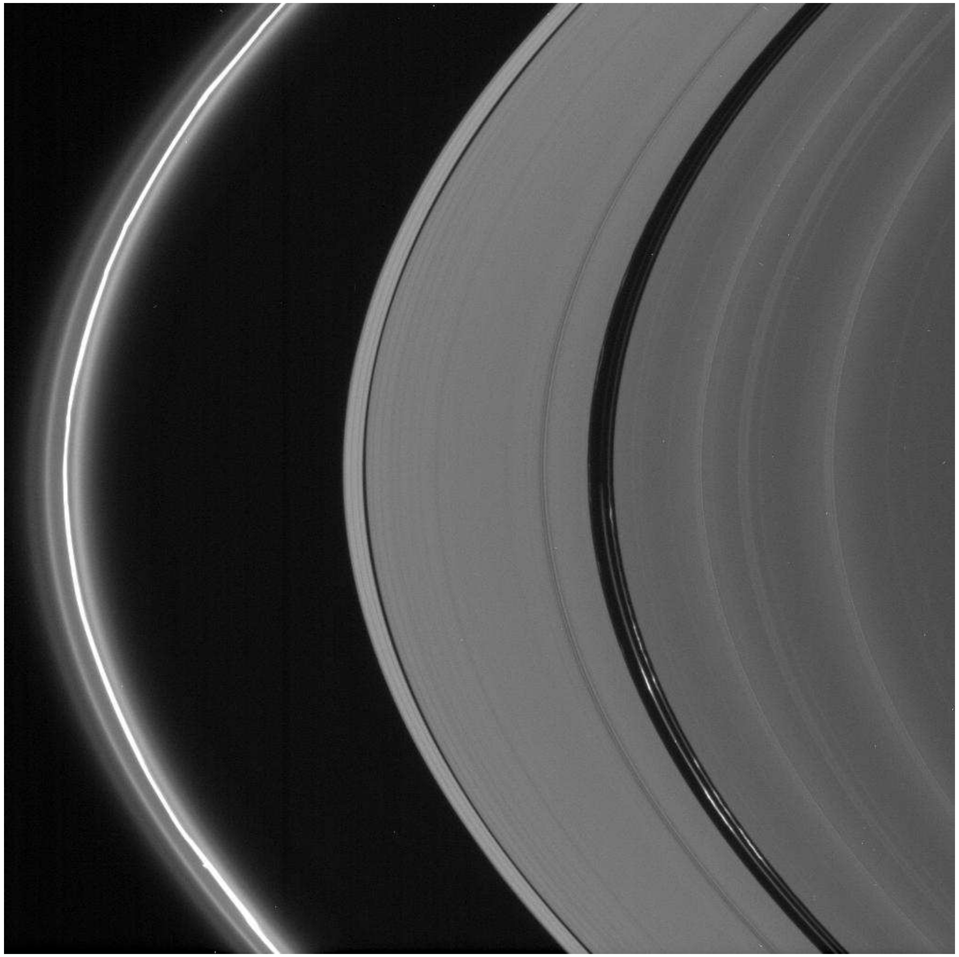
Saturn’s F ring, Prometheus and Pandora
PIA06143, PIA07523 (NASA/JPL/Space Science Institute)

Observations: narrow rings

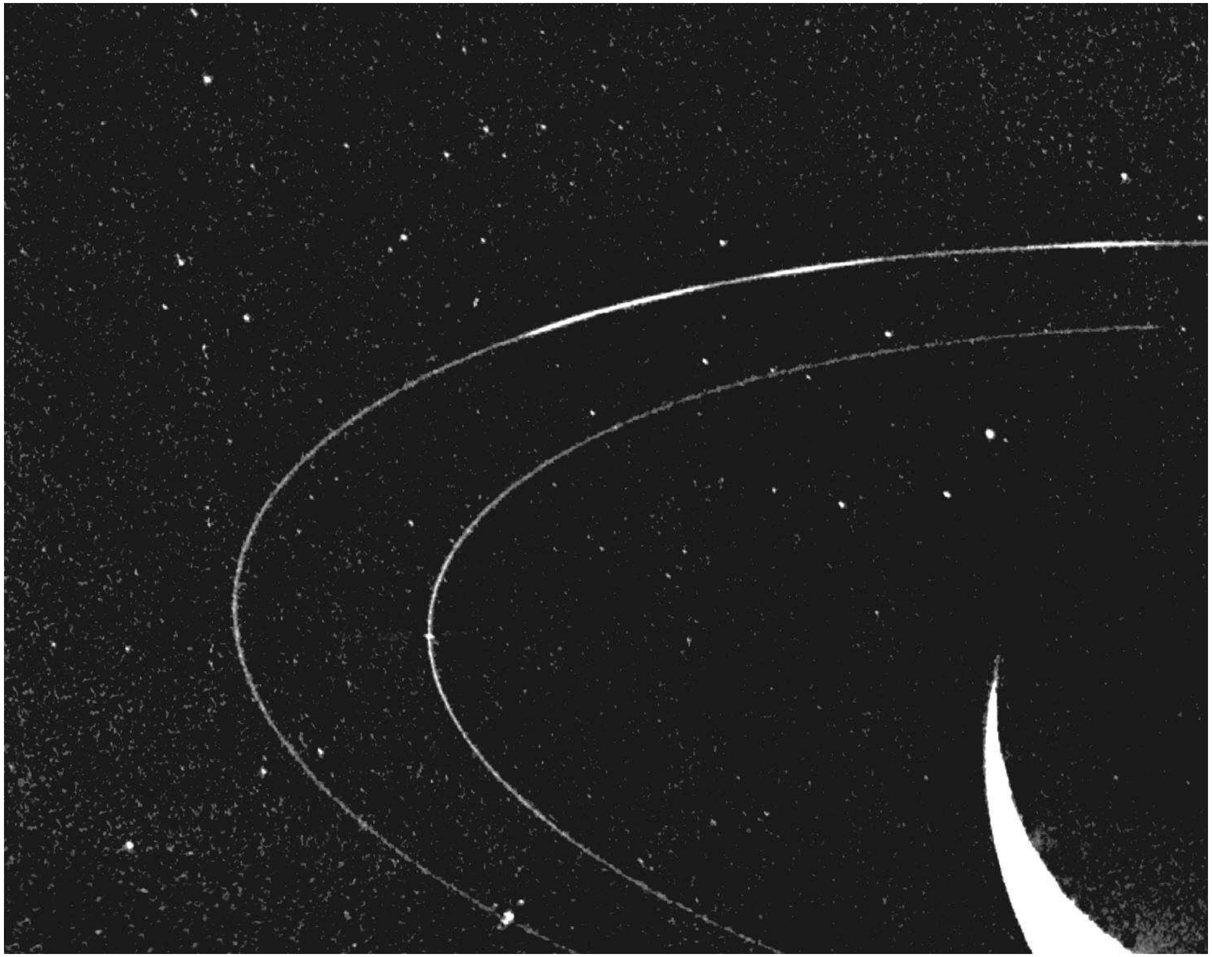


Structure in Saturn's F ring
PIA07522 (NASA/JPL/Space Science Institute)

Observations: narrow rings



Encke gap ringlets
PIA08305 (NASA/JPL/Space Science Institute)



Neptune rings and arcs
PIA01493 (NASA/JPL/Space Science Institute)

Open questions

We have a first-order understanding of the dynamics and key processes in rings, much of it based in previous work in galactic and stellar dynamics. (...) *Unfortunately, the models are often idealized (for example, treating all particles as hard spheres of the same size) and cannot yet predict many phenomena in the detail observed by spacecraft (for example, sharp edges). Non-intuitive collective effects give rise to unusual structures*.

(...) One such example is the case of shepherding satellites. The F ring is not exactly placed where the shepherding torques would balance. Of the Uranian rings, shepherds were found only for the largest ε (epsilon) ring; even so, they are too small to hold it in place for the age of the solar system. Another issue is that the sharp edges of rings are too sharp!

Larry Esposito, Planetary rings (Cambridge University Press, 2006).

Open questions

Some open issues are:

- Rings with sharp–edges, narrow and eccentricity
- Multiple ring components: Strands
- Clumps and arcs
- Kinks and bendings
- Stability, life times, origin, ...

Scattering approach to narrow rings

Consider the full $N+1$-Hamiltonian in an inertial frame, which can be written as ($N = N_{\rm moons} + N_{\rm ring\ particles}$)

$$\mathcal{H} = \sum_{i=0}^{N} \left[\frac{1}{2M_i} |\vec{P}_i|^2 - \frac{G M_0 M_i}{|\vec{R}_i - \vec{R}_0|} \right] - \sum_{i<j\neq 0}^{N} \frac{G M_i M_j}{|\vec{R}_i - \vec{R}_j|}$$

$$= \mathcal{H}_{\rm K_m} + \mathcal{V}_{\rm m-m} + \mathcal{H}_{\rm K_{rp}} + \mathcal{V}_{m-rp} + \mathcal{V}_{rp-rp}$$

Scattering approach to narrow rings

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1st approx.: no interaction among ring particles.

Consider the full $N+1$-Hamiltonian in an inertial frame, which can be written as ($N = N_{\rm moons} + N_{\rm ring\ particles}$)

$$\begin{aligned}\mathcal{H} &= \sum_{i=0}^{N}\left[\frac{1}{2M_i}|\vec{P}_i|^2 - \frac{GM_0M_i}{|\vec{R}_i - \vec{R}_0|}\right] - \sum_{i<j\neq 0}^{N}\frac{GM_iM_j}{|\vec{R}_i - \vec{R}_j|}\\ &= \mathcal{H}_{\rm K_m} + \mathcal{V}_{\rm m-m} + \mathcal{H}_{\rm K_{rp}} + \mathcal{V}_{m-rp}\end{aligned}$$

1st approx.: no interaction among ring particles.
2nd approx.: In the planetary case $M_{\rm rp} \ll M_{\rm m} \ll M_0$.
Thus, we replace the many-body problem by a collection of independent one–particle time-dependent Hamiltonians:

$$H = \frac{1}{2}|\vec{P}|^2 + V_0(|\vec{X}|, t) + V_{\rm eff}(|\vec{X}|, t)$$

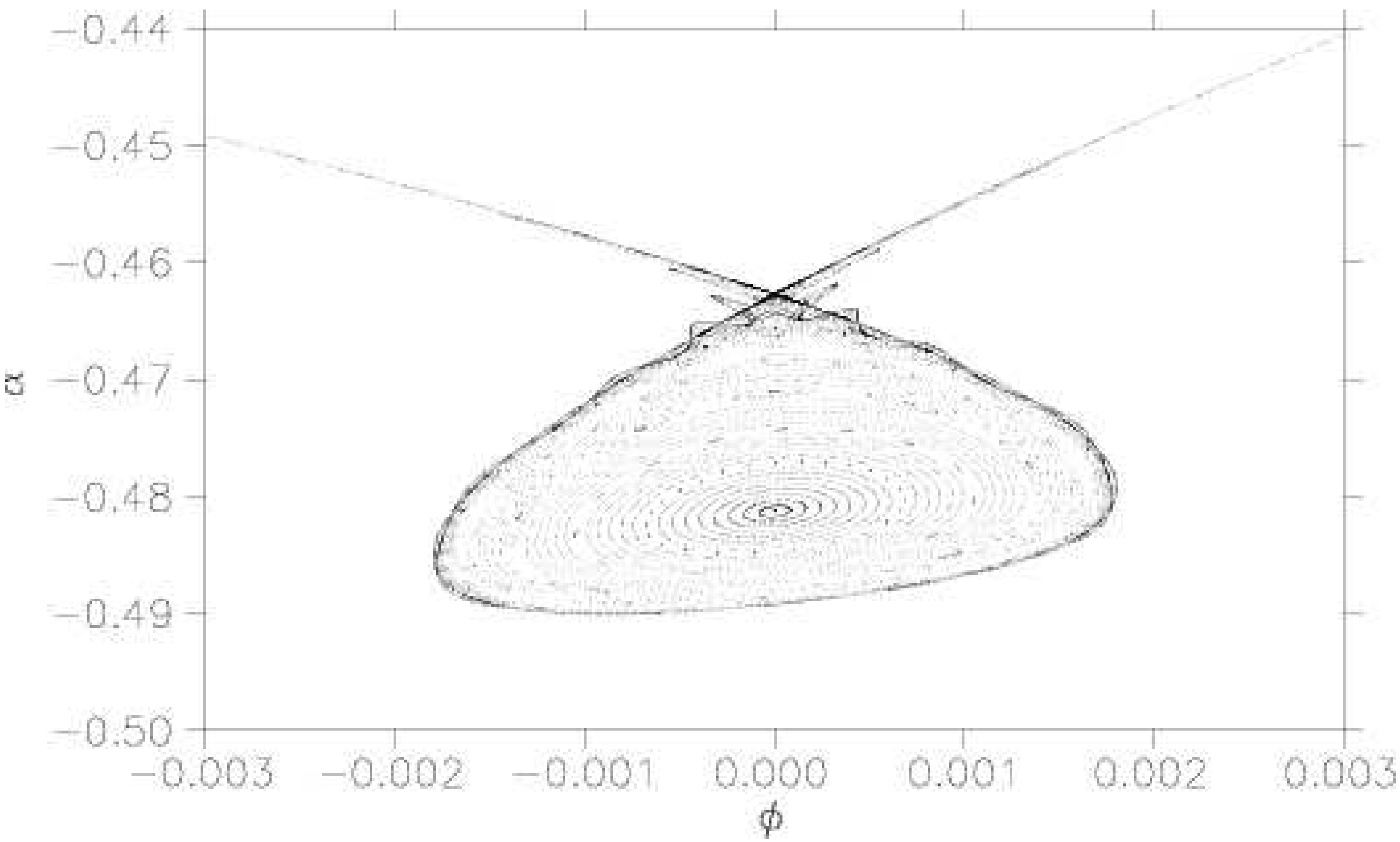
Restricted N-body problem $\Rightarrow$ *intrinsic rotation*

We shall concentrate on:

0. Intrinsic rotation
1. Phase–space regions where scattering dominates the dynamics: Escape to infinity is dominant
2. Organizing centers in phase space (periodic orbits or tori) are stable.
3. An ensemble of non–interacting particles with *almost-arbitrary* initial conditions

Rings are obtained by projecting onto the $X - Y$ space, at fixed time, *all dynamically trapped* particles

Phase space in a co-rotating frame



Consequences

Some *structural* consequences that follow from the assumptions:

- Scattering dynamics $\Rightarrow$ rings have sharp edges
- Orbits of organizing centers $\Rightarrow$ eccentric rings
- Small stable regions in phase space $\Rightarrow$ narrow rings

The whole scattering approach is robust

The rotating billiard

Ring particles evolution is given by

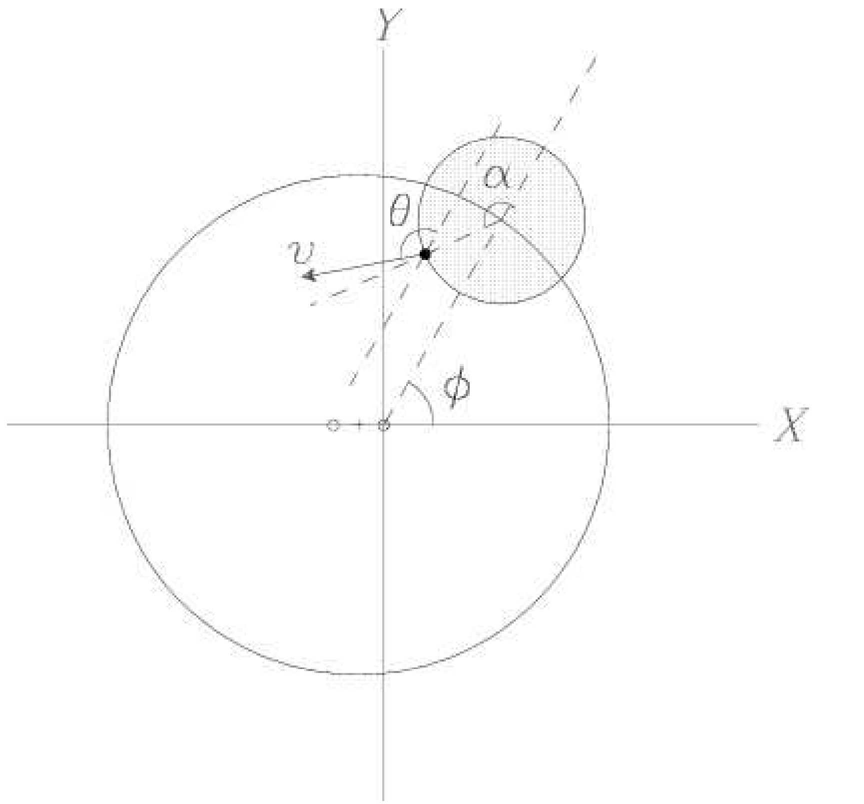
$$H = \frac{1}{2}|\vec{P}|^2 + V_0(|\vec{X}|, t) + V_{\mathrm{eff}}(|\vec{X}|, t)$$

The simplest case: planar billiard on a Kepler orbit

The rotating billiard

A hard-disk moving on a Kepler elliptic orbit

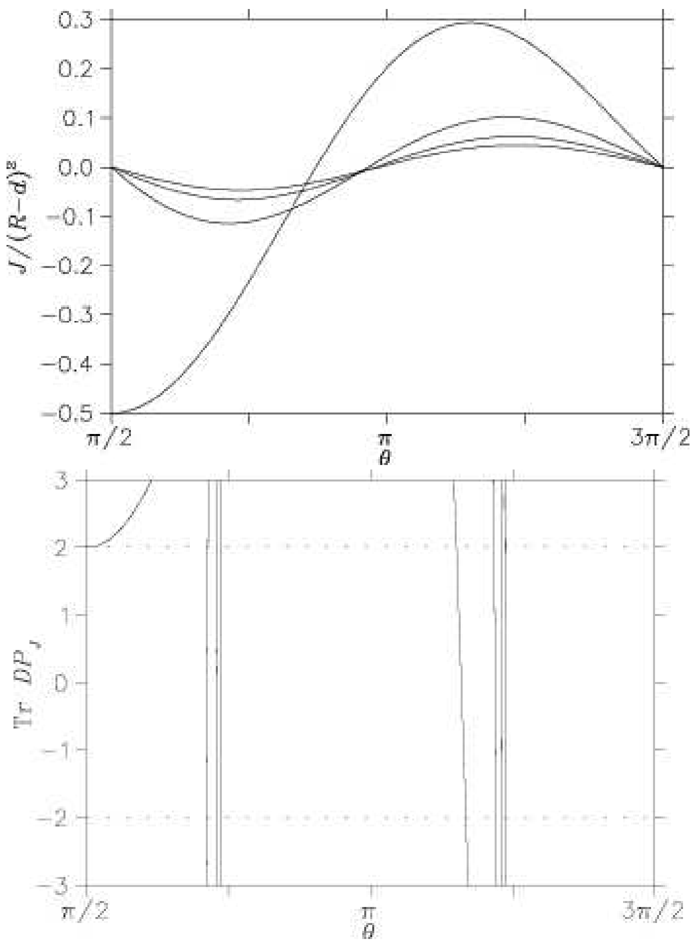
$$
\begin{aligned}
V_0(|\vec{X}|, t) &= 0 \\
V_{\text{eff}}(|\vec{X} - \vec{R}_d(t)| > d) &= 0 \\
V_{\text{eff}}(|\vec{X} - \vec{R}_d(t)| \le d) &= \infty
\end{aligned}
$$



$$
\begin{aligned}
R_d &= \frac{a(1-\varepsilon^2)}{1+\varepsilon \cos \phi} \\
R_d^2 \dot{\phi} &= a(1-\varepsilon^2)^{1/2}
\end{aligned}
$$

Simpler periodic orbits: Consecutive radial–collision orbits

Circular case: Periodic orbits and stability



Radial periodic orbits:

$$\frac{J_n}{(R-d)^2} = \frac{2\cos^2\theta + \Delta\phi\sin(2\theta)}{(\Delta\phi)^2}$$

with $\Delta\phi = (2n-1)\pi + 2\theta$

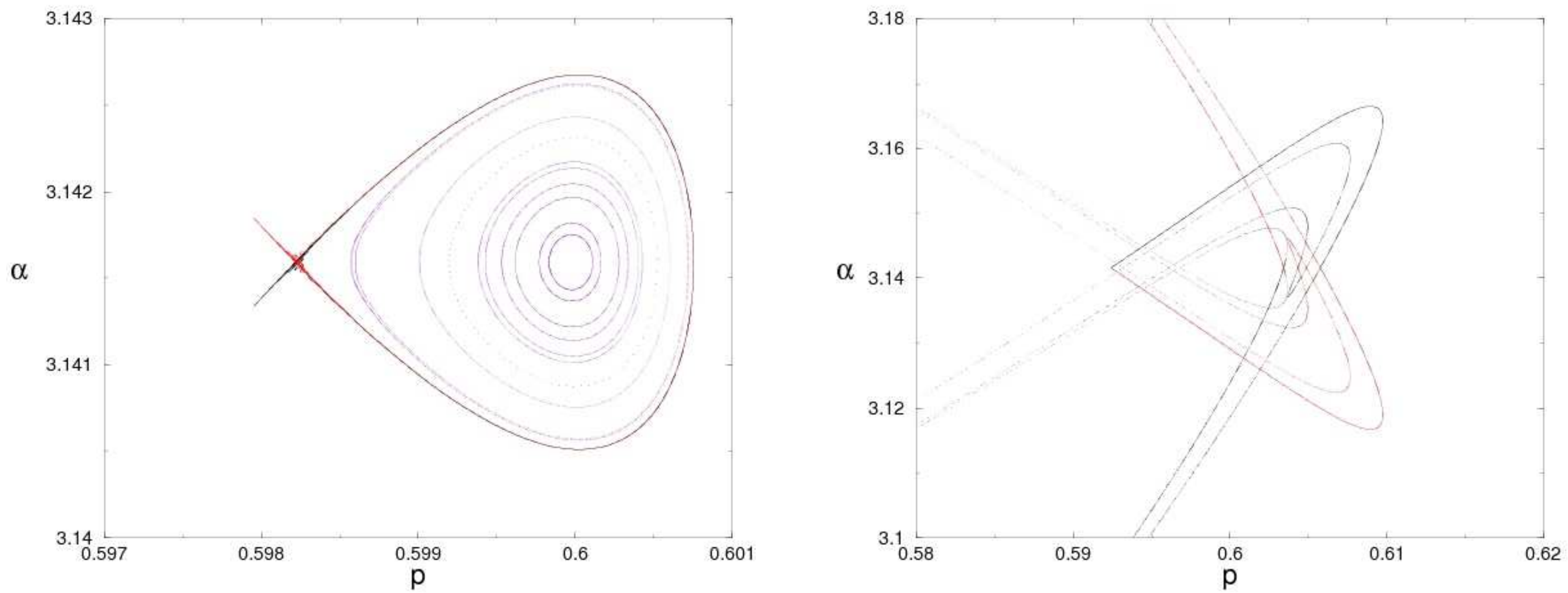
Stability:

$$\mathrm{Tr}\, D\mathcal{P}_J = 2 + \frac{(\Delta\phi)^2(1-\tan^2\theta)}{d/R} - \frac{4(1+\Delta\phi\tan\theta)}{d/R}$$

Changes of stability at $\mathrm{Tr}\, D\mathcal{P}_J = \pm 2$

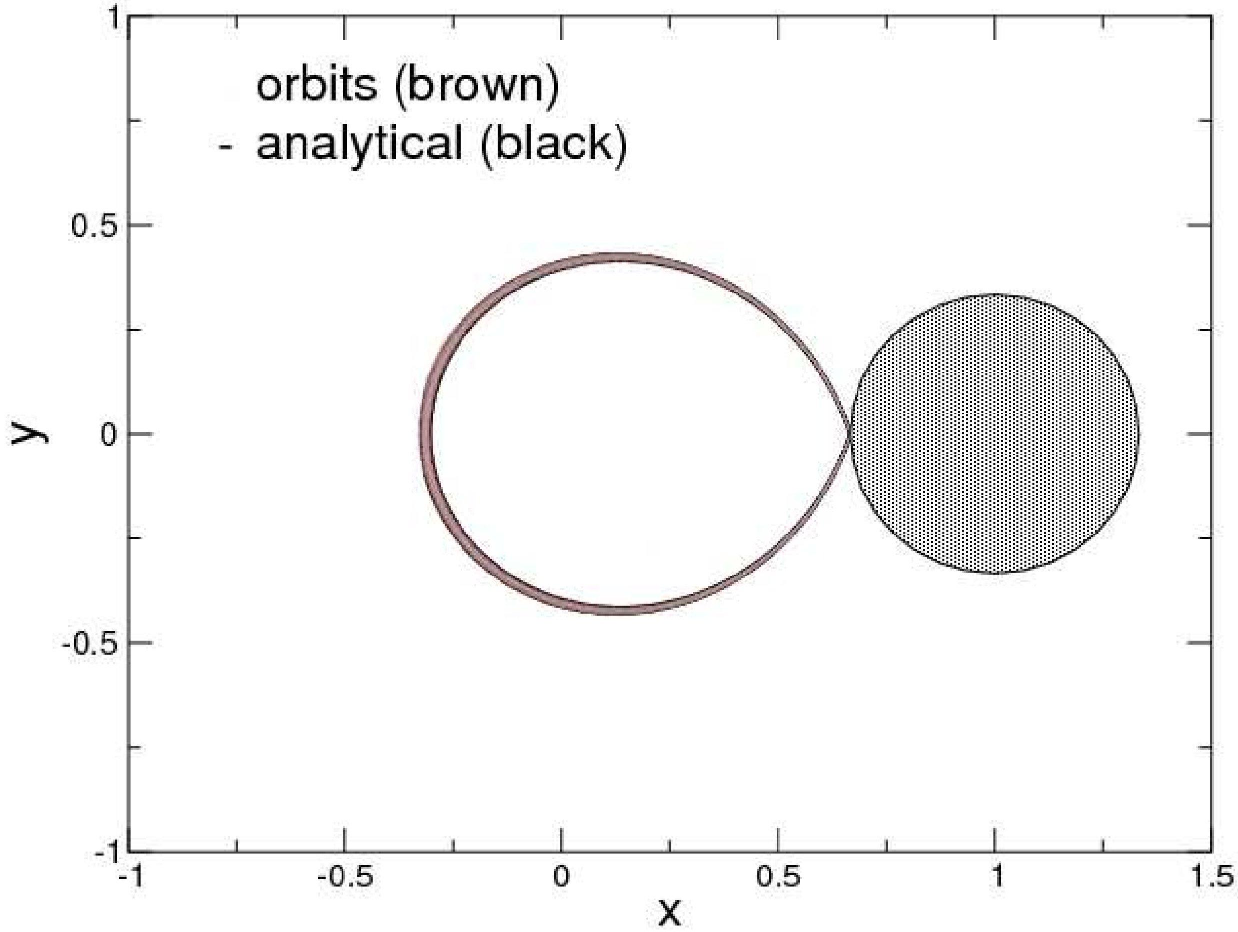
Circular case: Periodic orbits and stability

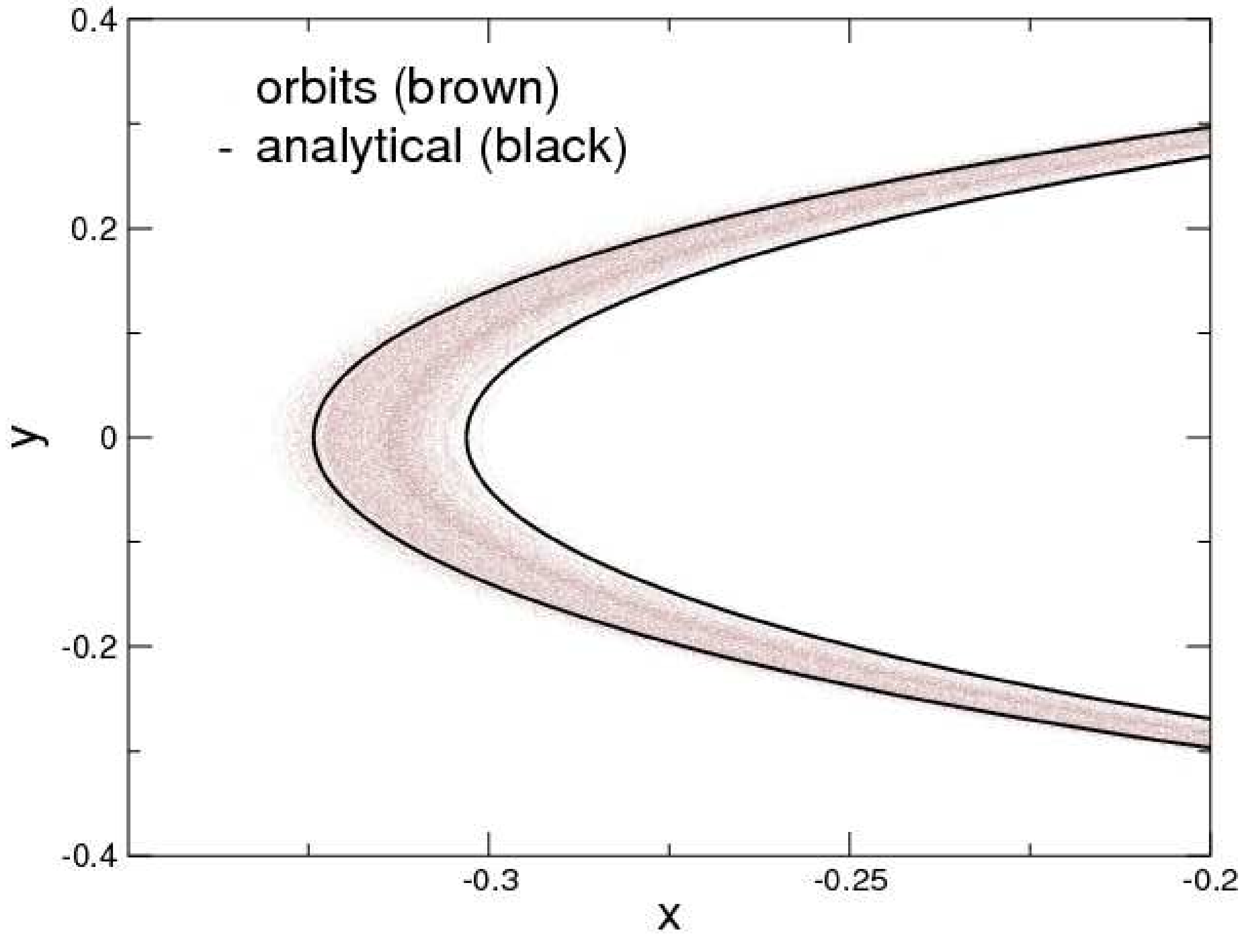
$$J/(r-d)^2 = 0.29325 \qquad J/(r-d)^2 = 0.29218$$



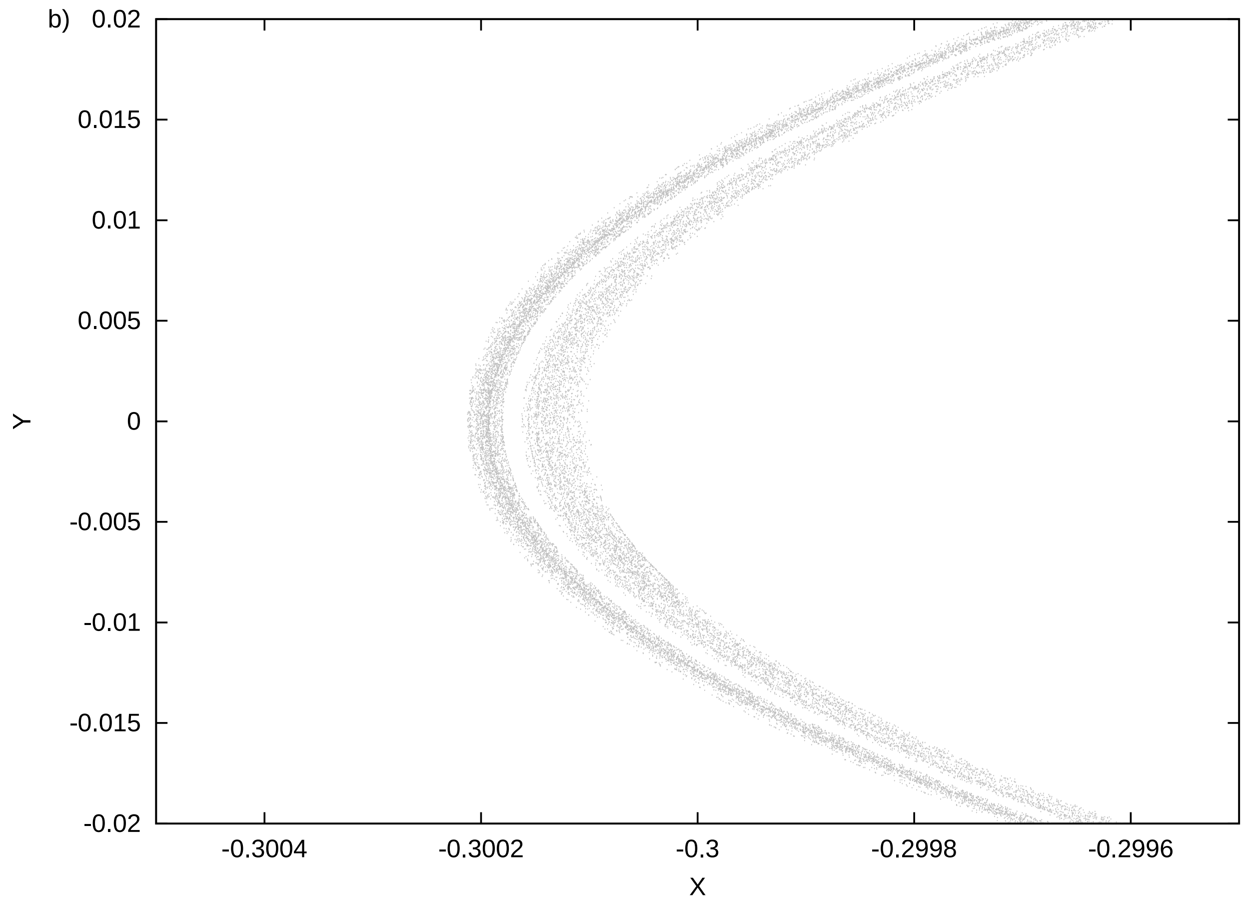
$$p = -d - R\cos\alpha - v\sin(\alpha-\theta)$$

Occurrence of rings

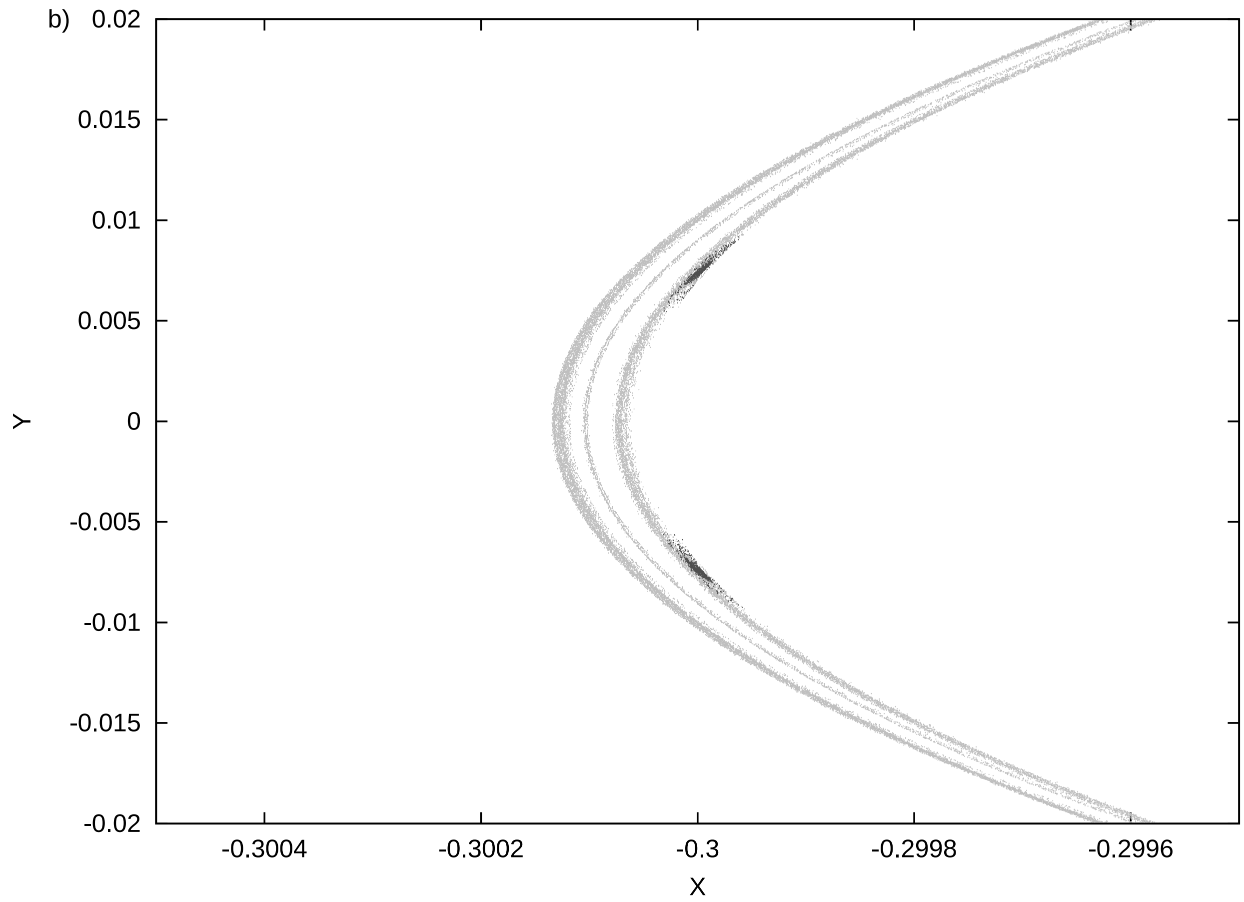




orbits (brown)
- analytical (black)
0.4
0.2
0
-0.2
-0.4
y
-0.3
-0.25
-0.2
x

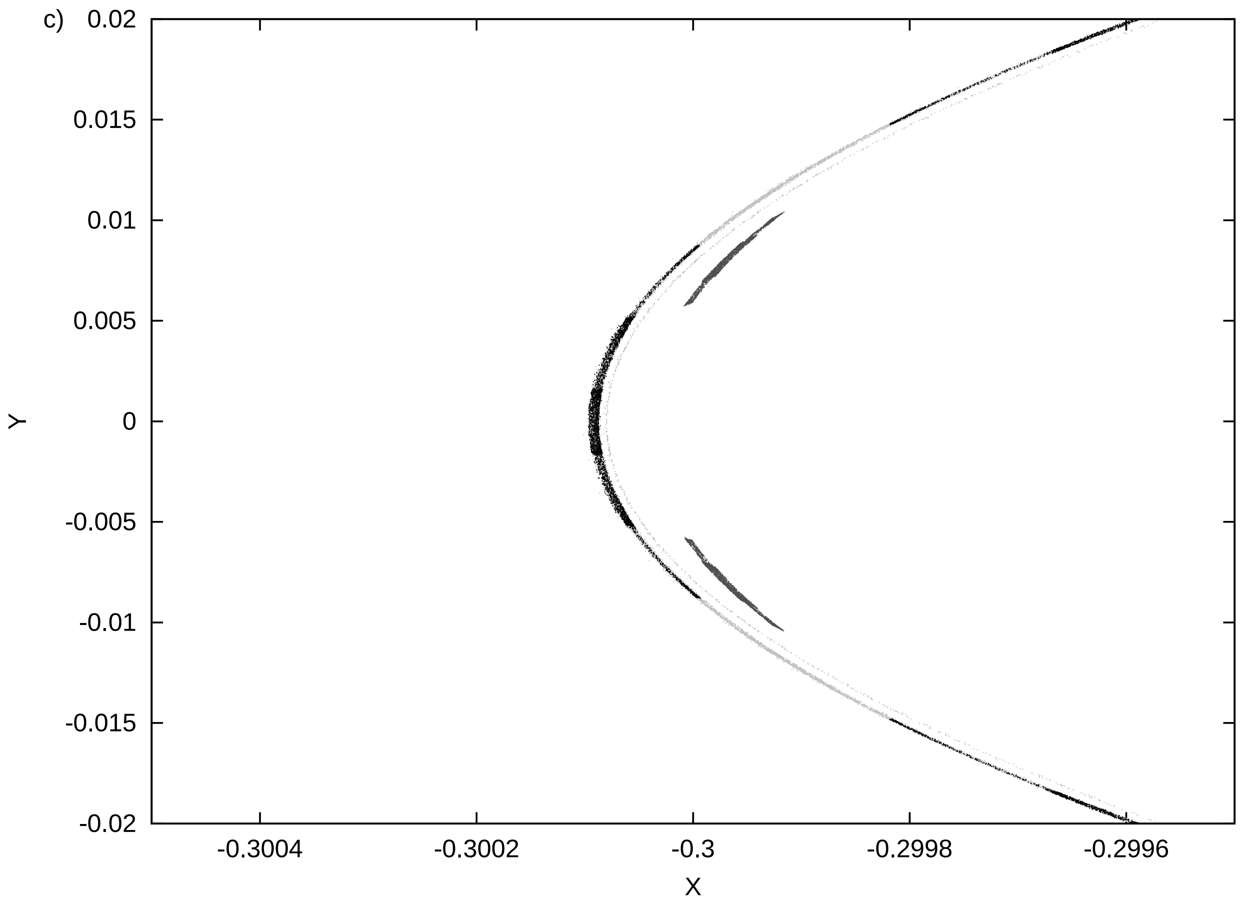


$$\varepsilon = 0.00165$$

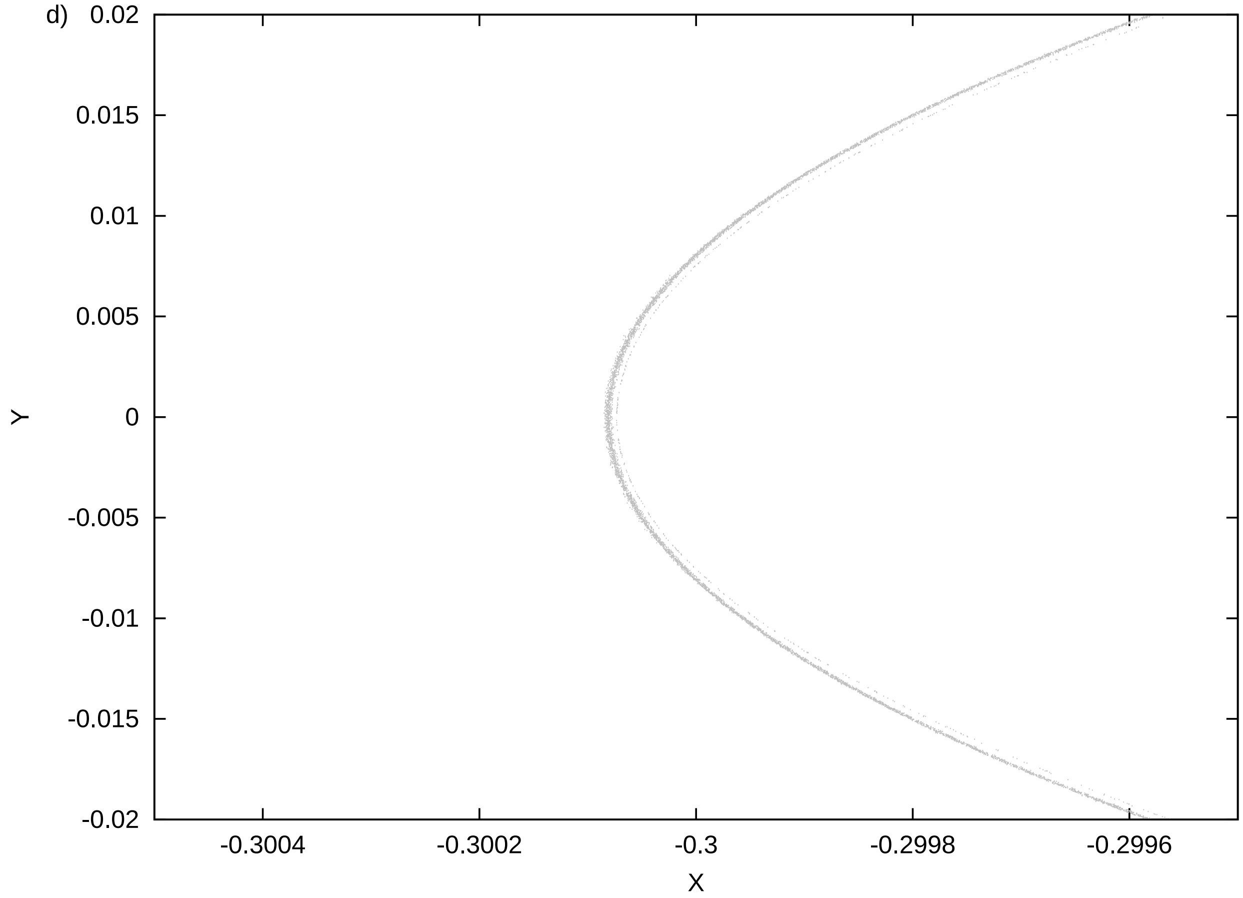


$$\varepsilon = 0.00167$$

Strands and Arcs



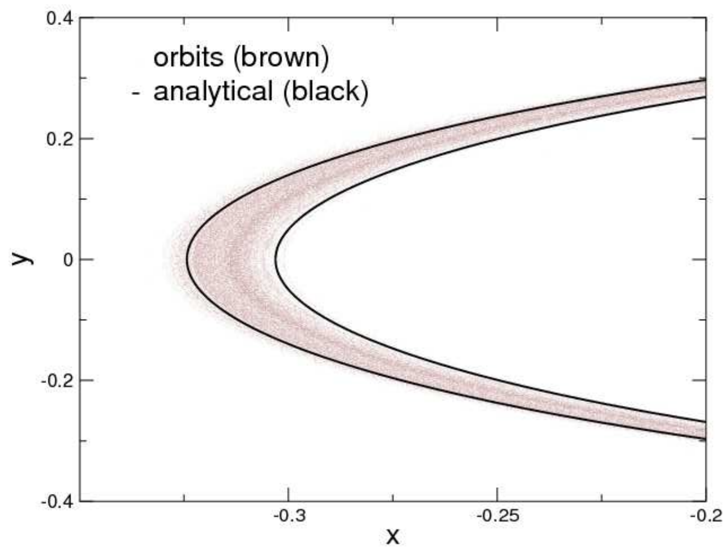
$$\varepsilon = 0.00168$$

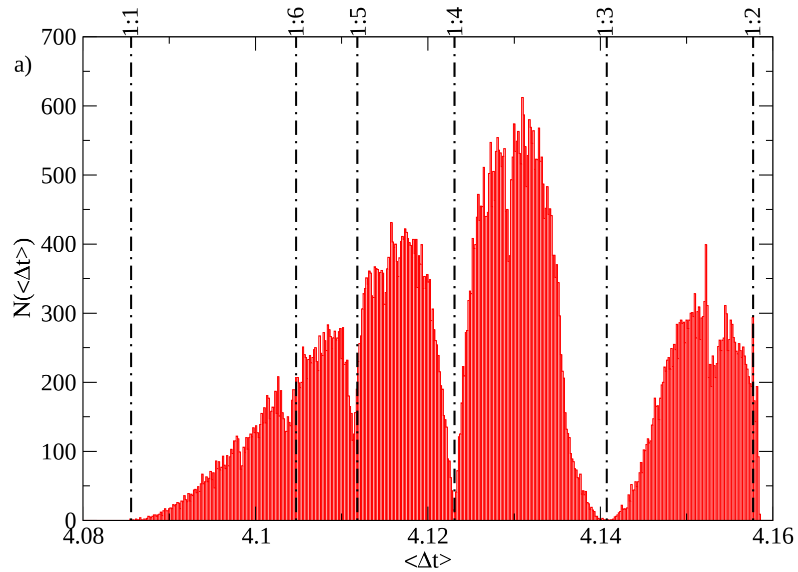


$$\varepsilon = 0.001683$$

Phase-space volume of trapped regions

$$\varepsilon = 0$$

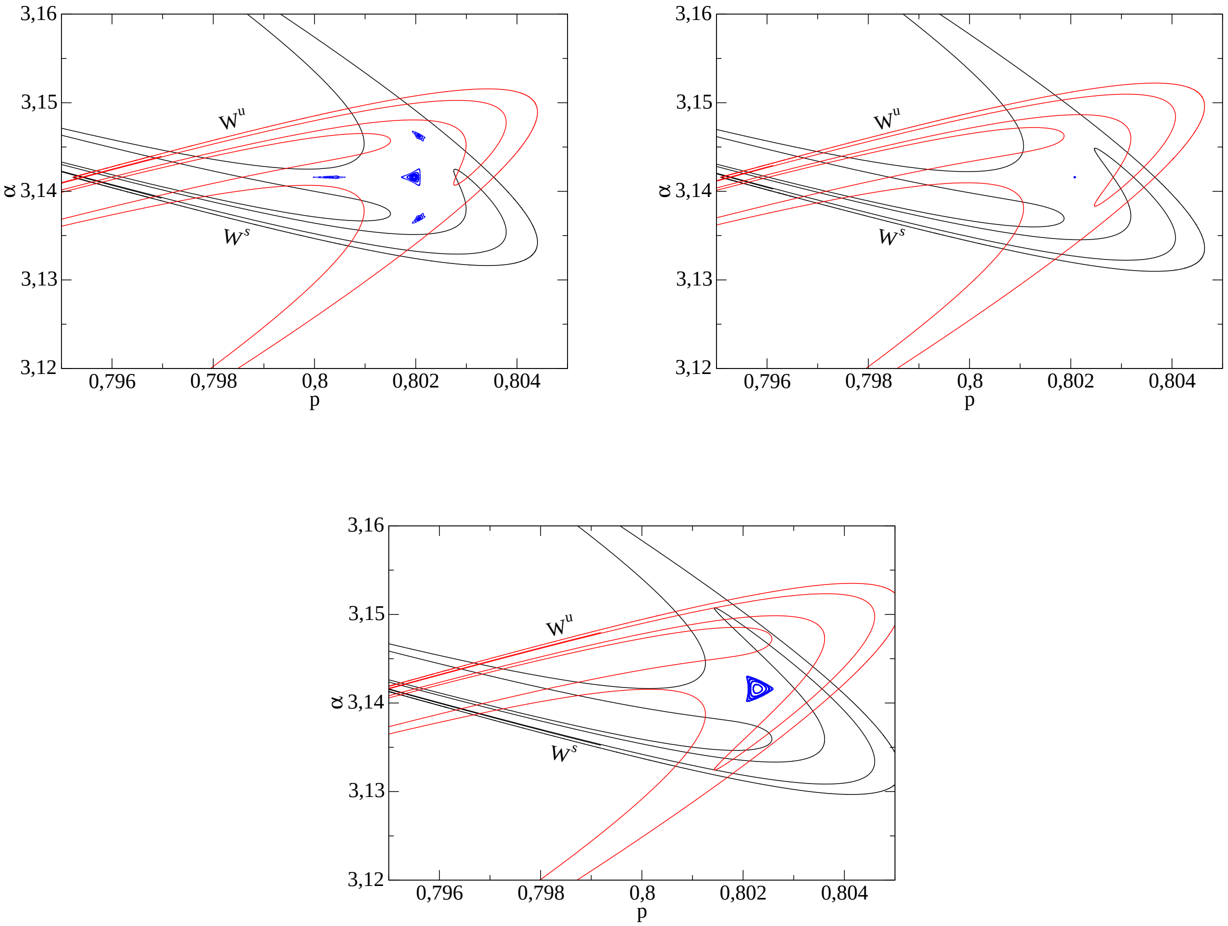




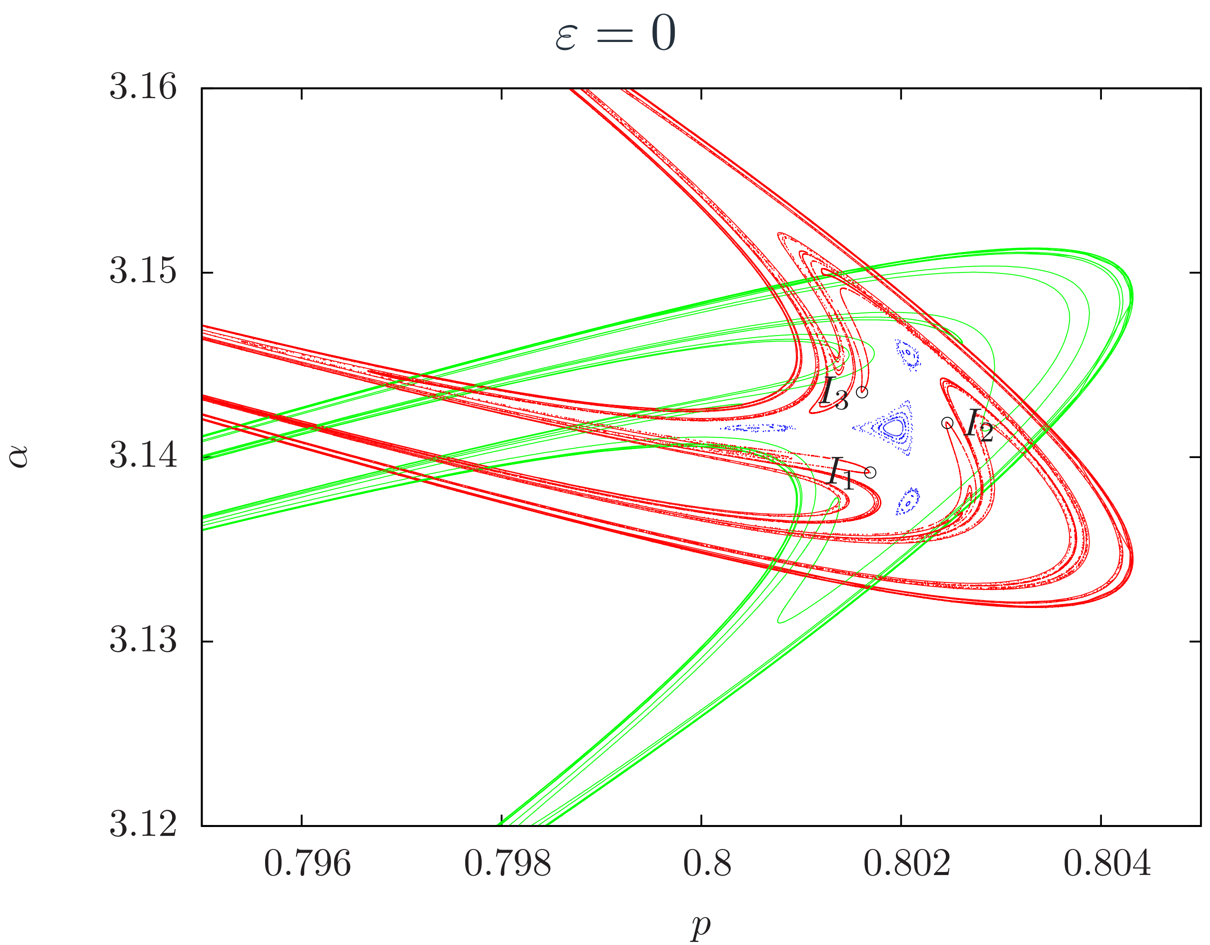
Stability resonance: $e^{i\alpha}$, $\cos(\alpha) = 2\mathrm{Tr}\, D\mathcal{P}_J$, $\alpha_{p:q}/(2\pi) = p/q$

Phase-space volume of trapped regions

$$\varepsilon = 0$$

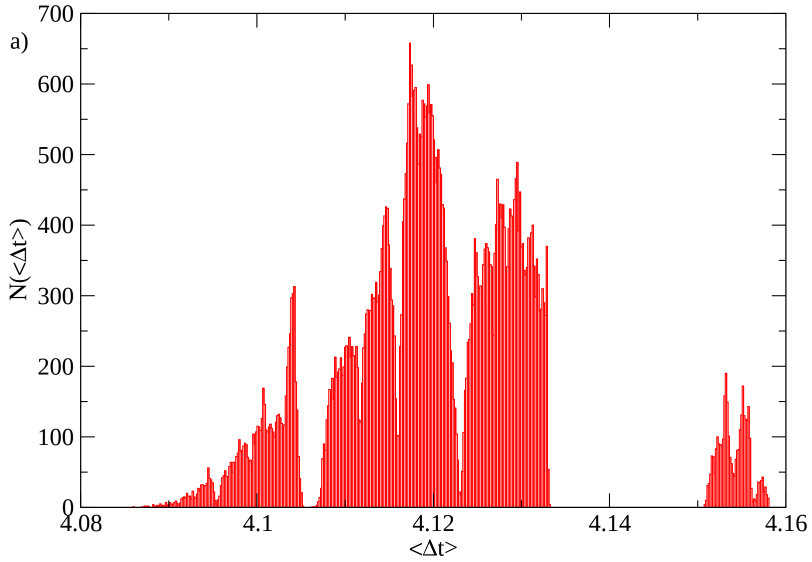


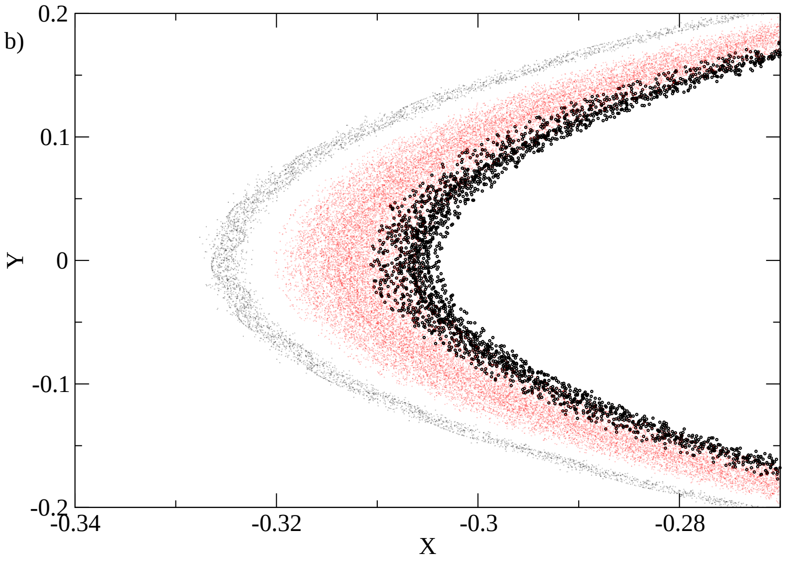
Phase-space volume of trapped regions



Phase-space volume of trapped regions

$$\varepsilon \neq 0$$

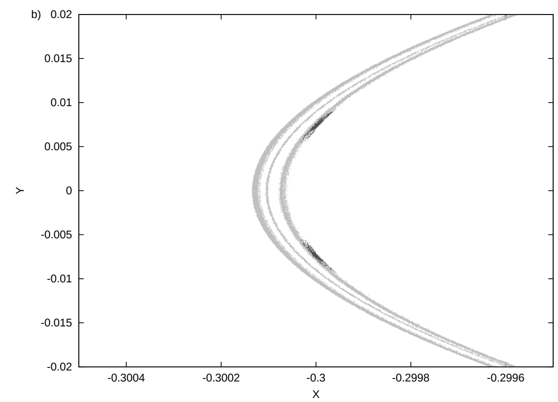




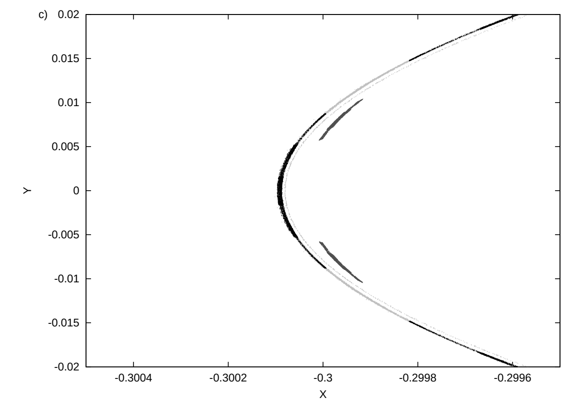
Excitation of stability resonances separates regions in phase space

Mean-motion Resonances

$\varepsilon = 0.00165$

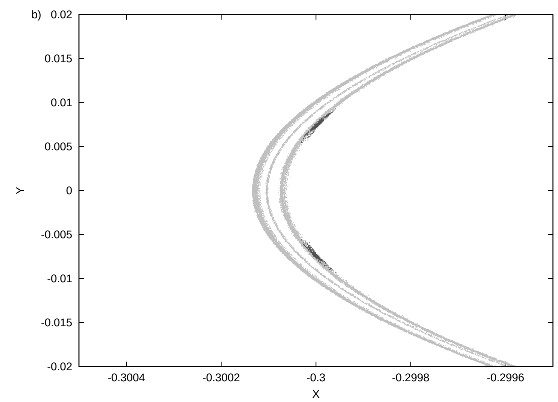


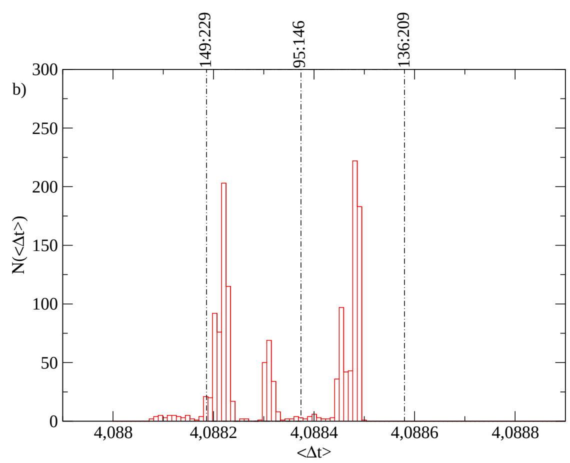
$\varepsilon = 0.00167$



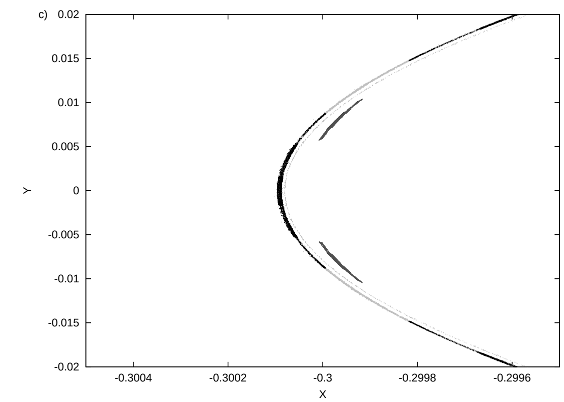
Mean-motion Resonances

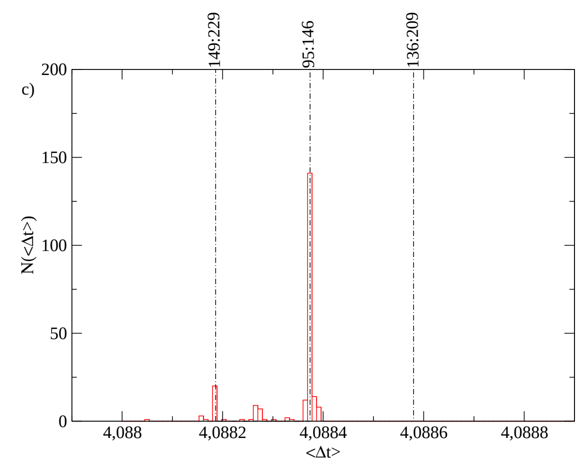
$\varepsilon = 0.00165$



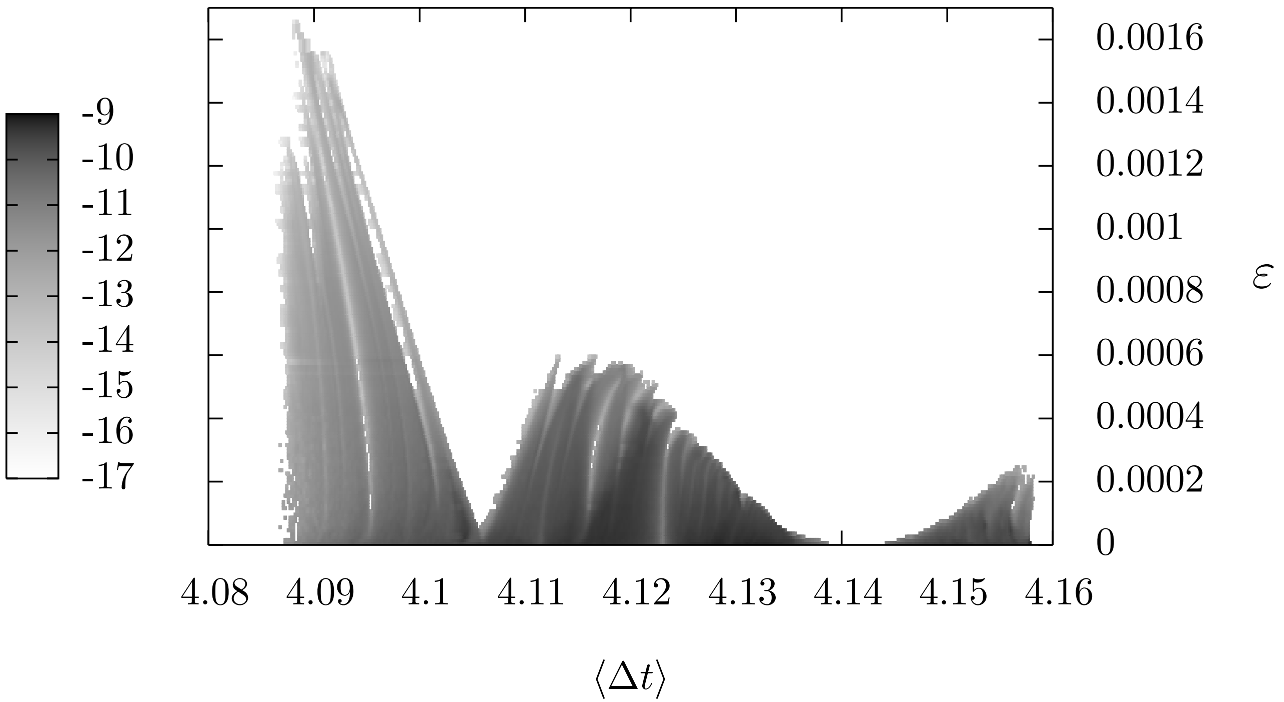


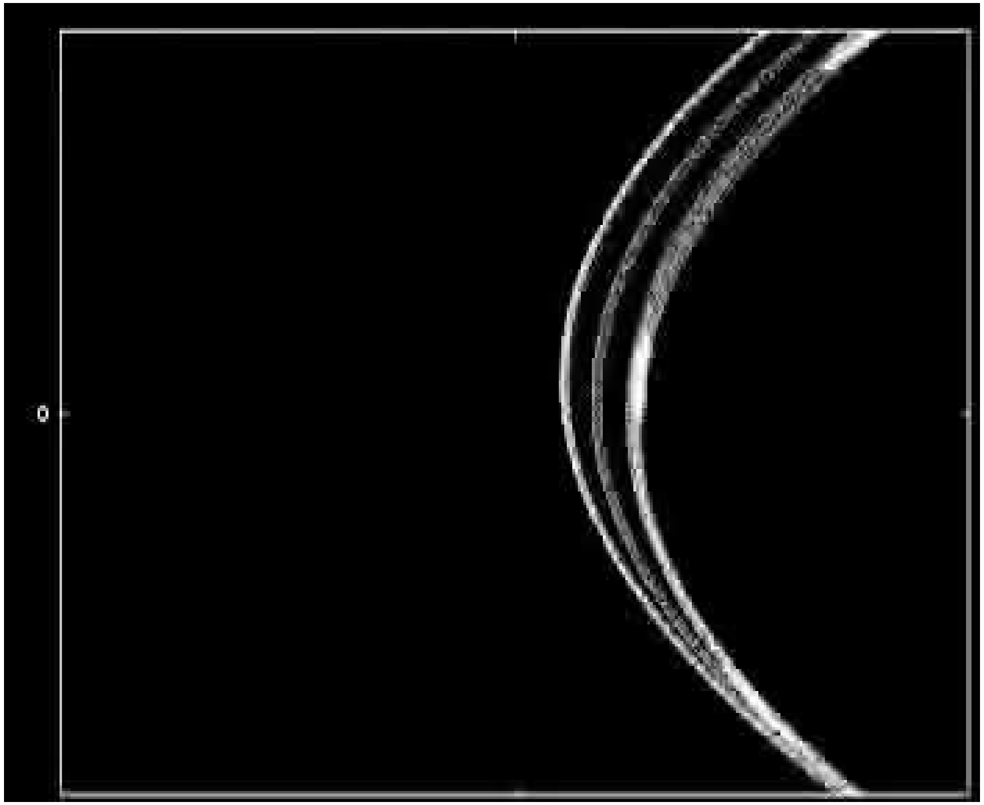
$\varepsilon = 0.00167$





Mean-motion Resonances

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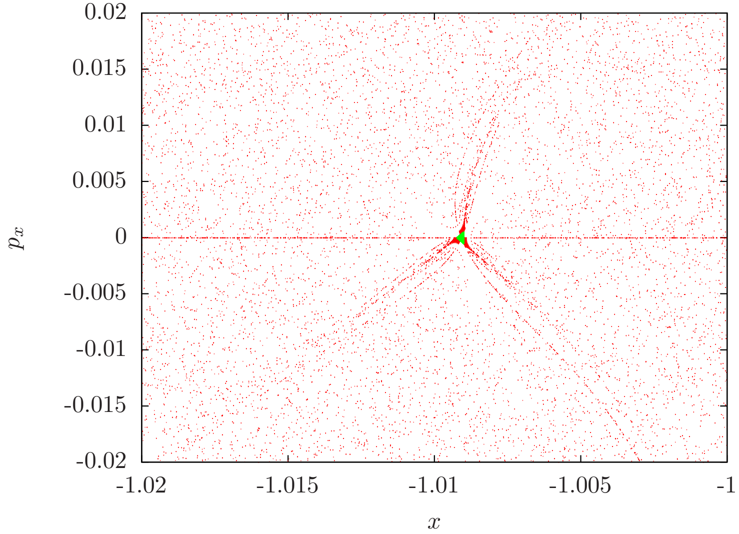
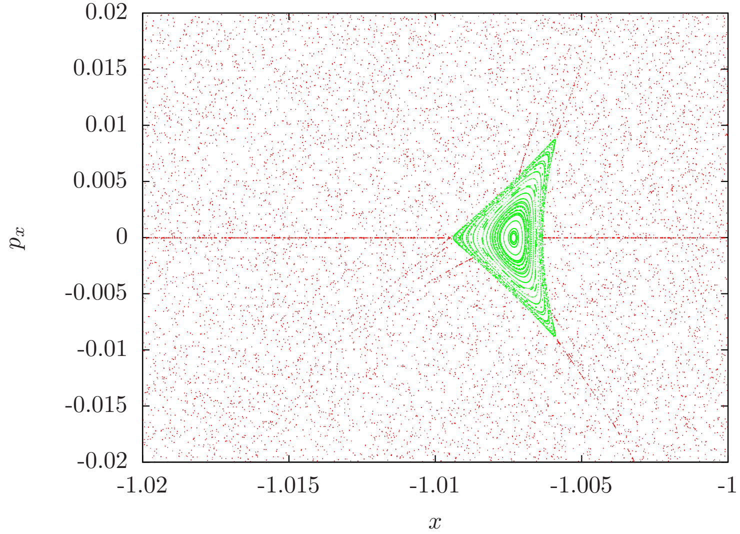
Summary

- Using a scattering obtain *consistently* sharp–edged narrow eccentric rings.
  Scattering dynamics $\Rightarrow$ rings with sharp edges
  Eccentric orbits as organizing centers $\Rightarrow$ eccentric rings
  Small stable regions in phase space $\Rightarrow$ narrow rings
- For more than two degrees of freedom, rings may have several components, strands. These appear by exciting certain *stability resonances* which lead to instabilities.
- Arcs (clumps) are related to mean-motion resonances within thin strands.

Outlook (work in progress)

CRTBP $\mu = 2.086 \times 10^{-6}$





Outlook (work in progress)

Ring obtained using a realistic *consistent* gravitational restricted 5 body problem