Projecte MTM 2021-2023

Dinàmica complexa

Holomorphic Days
Castelldefels, 12-13 Juliol 2021

Universitat de Barcelona
URV Universitat Rovira i Virgili
UAB Universitat Autònoma de Barcelona
UJI Universitat Jaume I

CRM Centre de Recerca Matemàtica
BGSMath Barcelona Graduate School of Mathematics

PROJECTE

LINE 1: **Combinatorial and Topological Dynamics (UAB, UdG, UJI)**

1.5 -- Topological dynamics and rotation theory for the circle (Al - Ca)

LINE 2: **Dynamical systems and Applications: Biology and Astrodynamics (UAB, UdG, URV)**

2.4 -- Ghosts from the complex: holomorphic dynamics after a fold bifurcation (Al - Ca - Fa)

LINE 3: Qualitative Theory of differential Equations (UAB, UdG, URV)

LINE 4: **Dynamics of functions with essential singularities (UB)**

4.1 -- Non-preperiodic stable components and non-autonomous dynamical systems (Fa - Ja)

4.2 -- Topology and dynamics of invariant sets (Fa - Ja)

4.3 -- Transcendental functions of finite type: dynamical and parameter plane (Fa - Ja - Pa)

LINE 5: **Real and complex rational dynamical systems (UB, URV, UJI)**

5.1 -- Numerical methods viewed as dynamical systems (Ca - Ga - Ja - Pa)

5.2 -- Singular perturbations (Ca - Ga - Ja- Pa)

5.3 -- Invariant objects under rational iteration (Ca - Fa - Ja - Ga)

LINE 6: **Bifurcation diagrams in differential equations (URV)**

6.3 -- The separatrix graph of a rational vector field (Ga)

6.4 -- Bifurcations in rational vector fields (Ga - Vi)

LINE 1: **Combinatorial and Topological Dynamics** (UAB, UdG, UJI)

1.5 -- Topological dynamics and rotation theory for circle maps (Alsedà + Canela)

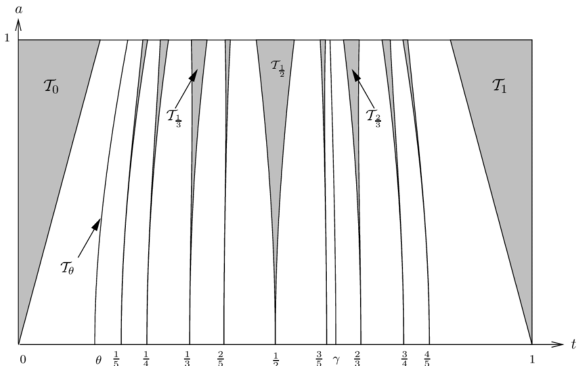
Notorious families of circle maps:

$$F_{a,b}(\theta) = \theta + a + b\sin(\theta) \pmod{2\pi}$$ (Arnold family, degree 1)

$$F_{a,b}(\theta) = 2\theta + a + b\sin(\theta) \pmod{2\pi}$$ (Double standard, deg 2)

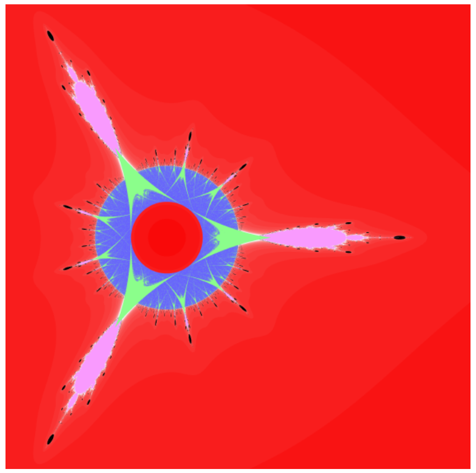
Thesis Jordi Canela

- Blaschke family analogous to double standard
- Rotation numbers and tongues extended to degree >1
- Complexification
- Relation to attracting periodic orbits

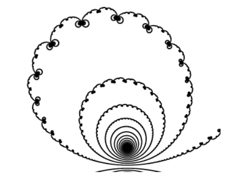


Project:

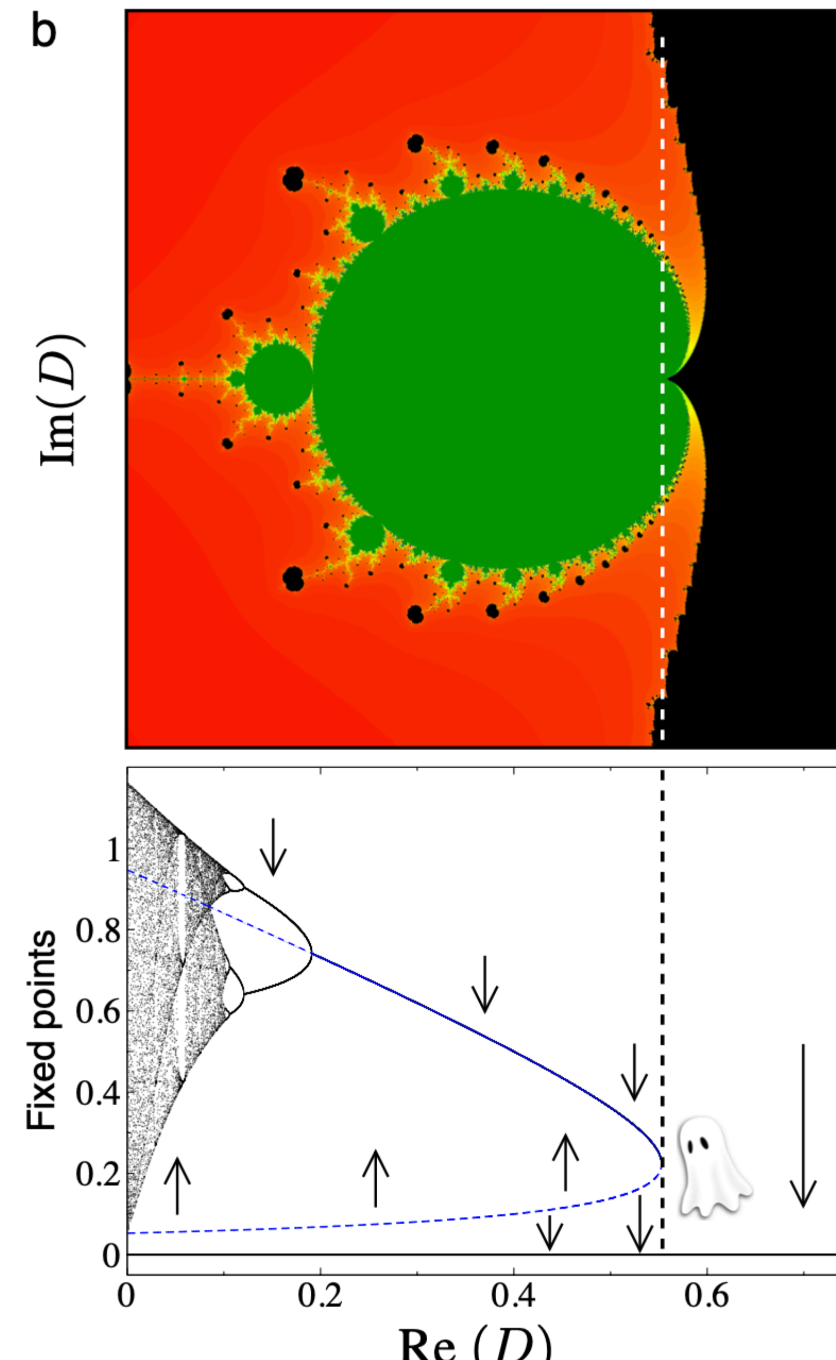
- Well-ordered attracting periodic orbits
- Tongues of well-ordered orbits
- Quasi-conformal surgery



2.4 -- Ghosts from the complex: holomorphic dynamics after a fold bifurcation



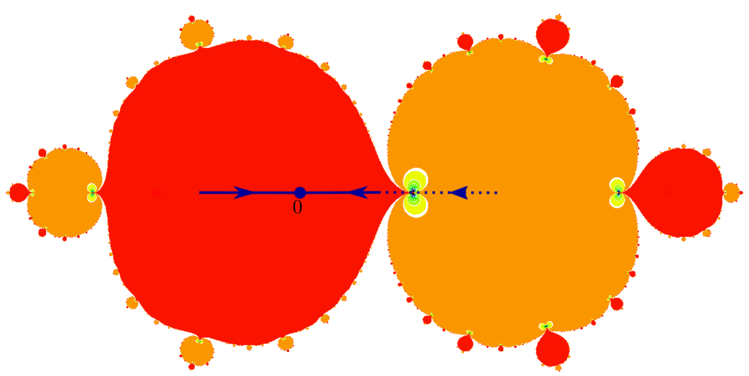
(Alsedà+Canela+Fagella+Sardanyés)



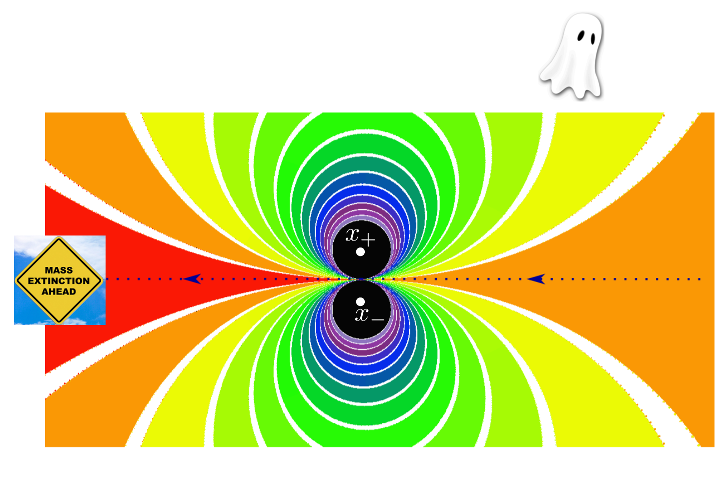
Ecological system with facilitation

$$F(z) = z + \mu z^2(1 - D - z) - \gamma z, \quad z \in \mathbb{C}$$

Analysis of a saddle node bifurcation from the complex ---> ghost effect



$D = D_c + 10^{-6}$



4.1 -- Non-preperiodic stable components and non-autonomous dynamical systems

(Fagella+Jarque+Lazebnik ---- Benini+Evdoridou+Fagella+Rippon+Stallard -- Florido)

- This is about **WANDERING DOMAINS**
  - If f is transcendental and U is a WD, let

$$L(U) = \{a \in \hat{C} \mid f^{n_k} \rightrightarrows a \text{ for some } n_k \to \infty\}$$

Conjecture 1: L(U) must contain a critical or asymptotic value of f

Conjecture 2: L(U) must contain the point at ∞

Goal 1: Construct a meromorphic example to show that Conj.1 is false for this class (quasiconformal folding)

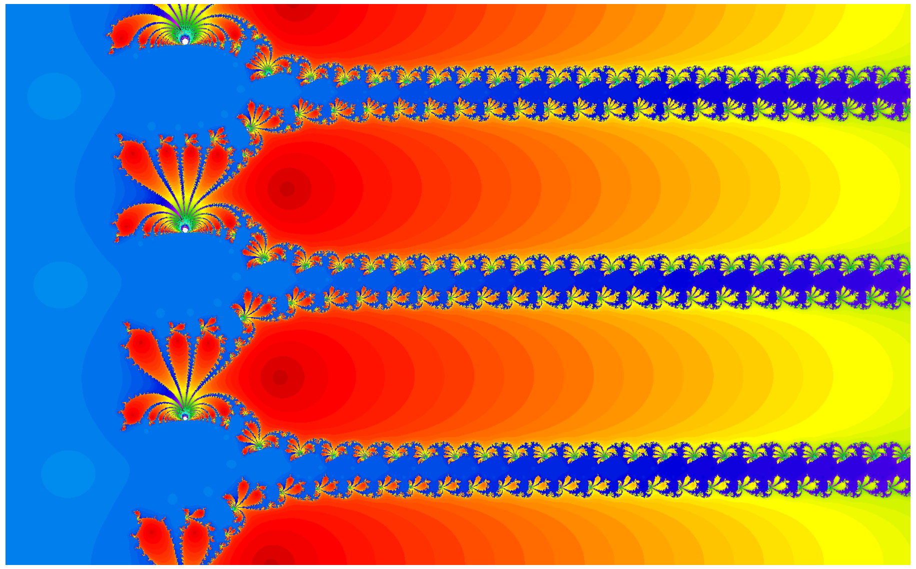
Goal 2: Study dynamics in the interior and on the boundary of WD, via sequences of inner functions.

4.1 -- Topology and dynamics of invariant sets

Baranski + Fagella + Jarque + Karpinska

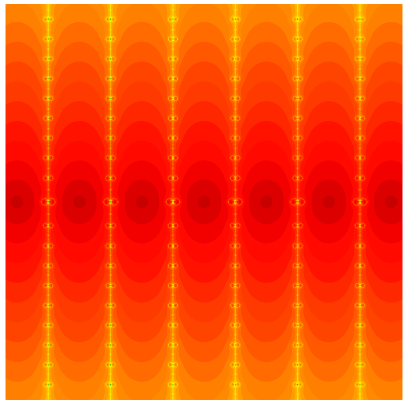
- We consider very general meromorphic functions (infinitely many singular values), but **TAME** at infinity. (Many Newton's methods!)

- We study the local connectivity of the boundary of invariant Fatou components and also the **local connectivity** of the Julia set



Conjecture: U invariant, unbounded, "tame" near infinity, then the boundary is L.C.

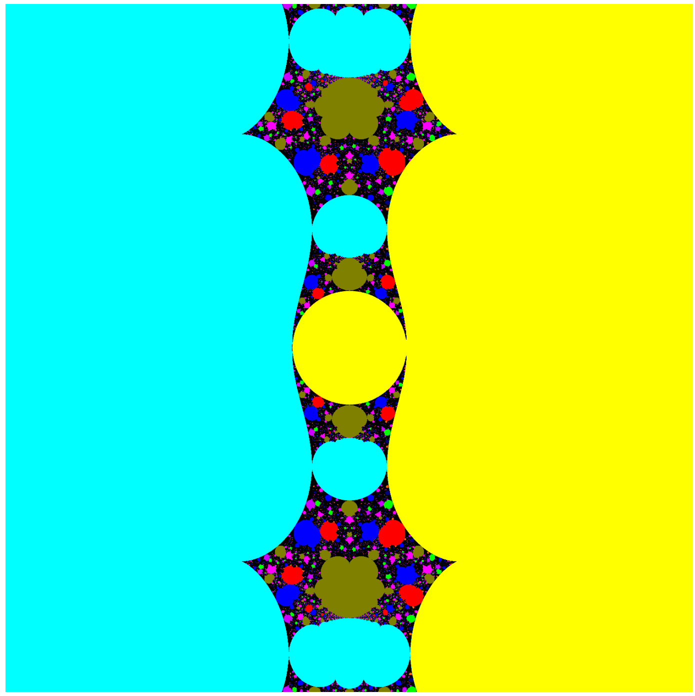
Goal: Prove the conjecture for attracting or parabolic basins, and for Baker domains of **finite degree.**



Tools / Steps:
1. Construction of a metric expanding on a neighborhood of the Julia set.
2. Deal with accesses to infinity
3. Whyburn's theorem for global Local Connectivity

4.3 -- Transcendental functions of finite type: dynamical and parameter plane

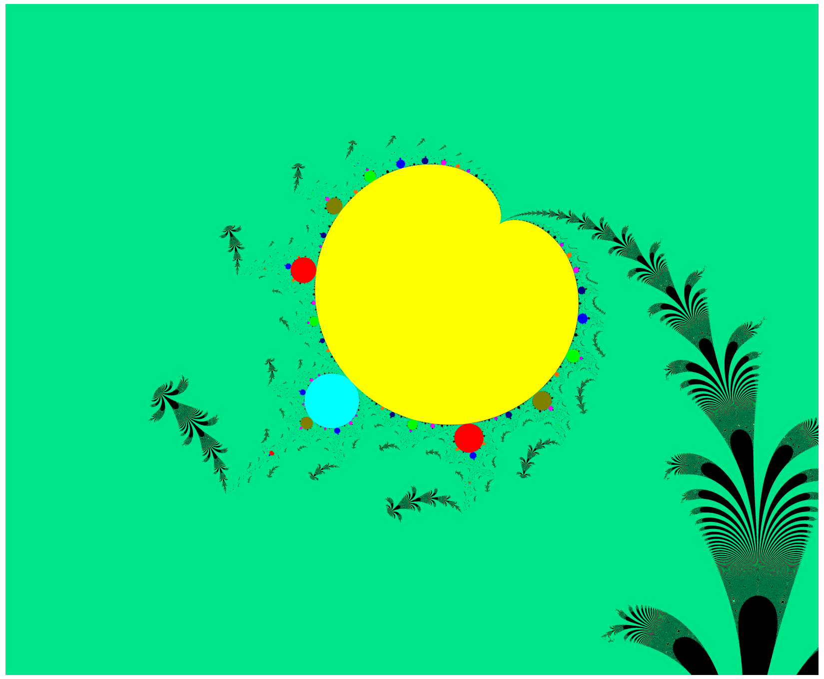
Astorg + Benini + Fagella --- Florido, Paraschiv?



- Finite type ---> finite number of singular values
- We study the PARAMETER PLANE of finite type families

$$\{\varphi_\lambda \circ f \circ \psi_\lambda\}_{\lambda \in M}, \quad \varphi_\lambda, \psi_\lambda \text{qc homeos}$$

- If f is rational or entire, **J-stable parameters** are dense in bif. locus [MSS, EL]

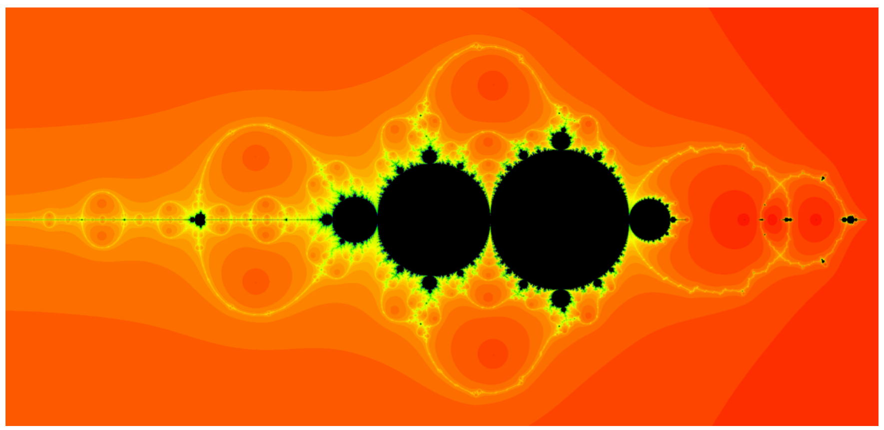


Theorems:

1. J-stable parameters are dense
2. Virtual centers iff singular prepoles

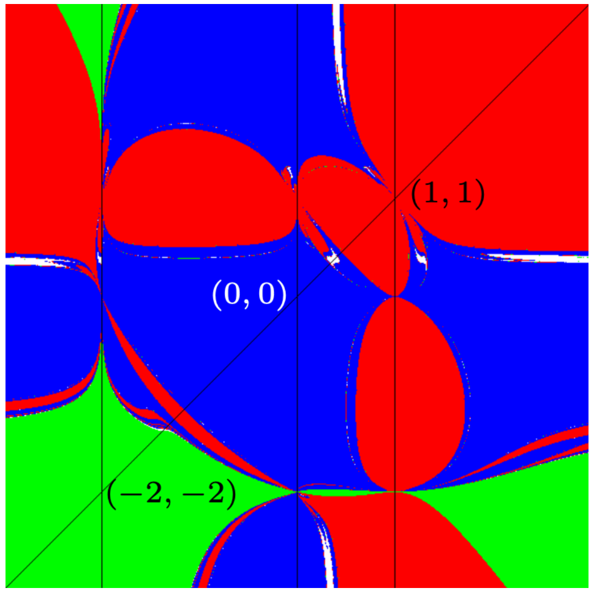
5.1 -- Numerical methods viewed as dynamical systems

Canela + Garijo + Jarque + Paraschiv + Vindel + Campos



- NEWTON'S METHOD: every Fatou component is simply connected.

- Other interesting methods are:
  -- Traub's method
  -- Chebyshev-Halley's method
  -- Secant's method (real plane)

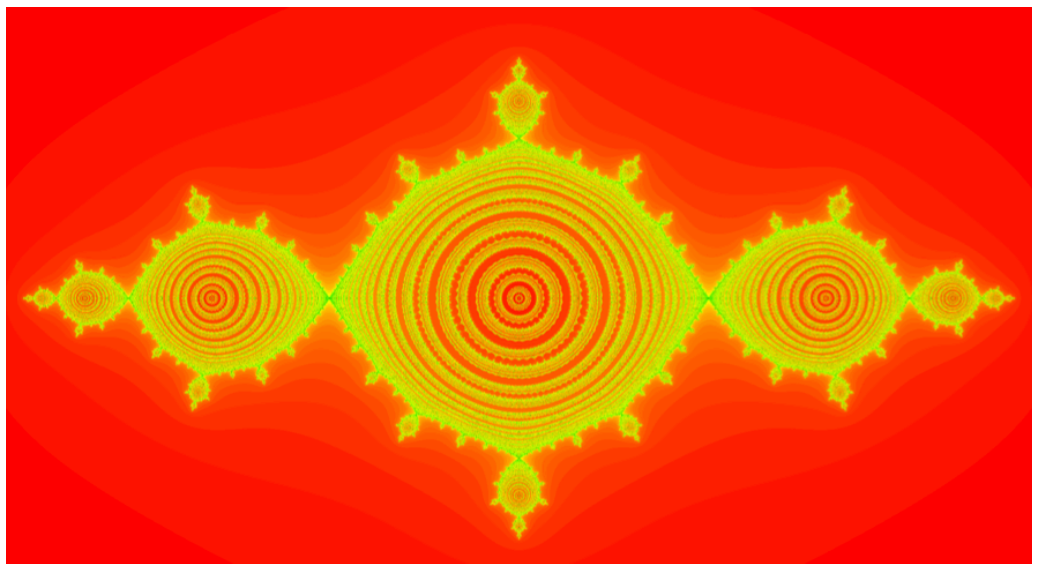


Questions:

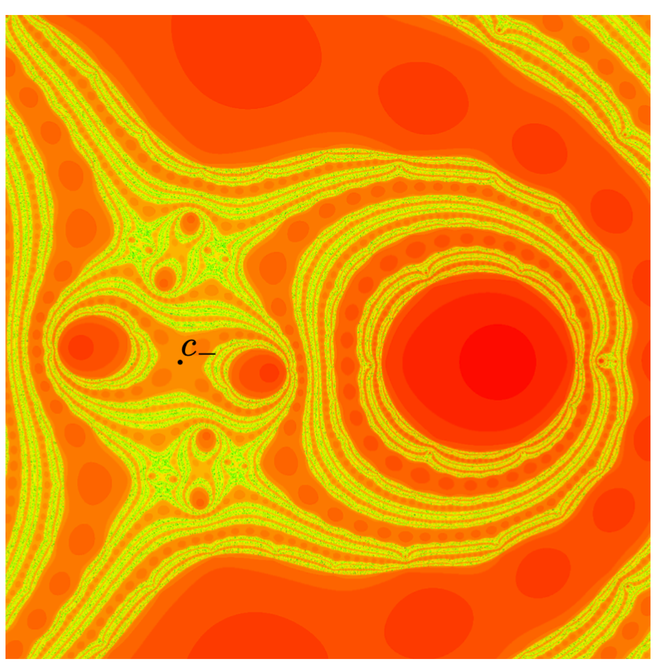
1. Structure of dynamical and parameter planes
2. **Simple connectivity** of attracting basins
3. Hyperbolic components in paramter plane
4. Numerical applications?

5.2 -- Singular perturbations

Canela + Garijo + Jarque + Paraschiv



- Singular perturbations are perturbations which change the function's nature (e.g. adding a pole) [Devaney et al]
- Canela'18 proved the existence of Fatou components of **arbitrary connectivity**



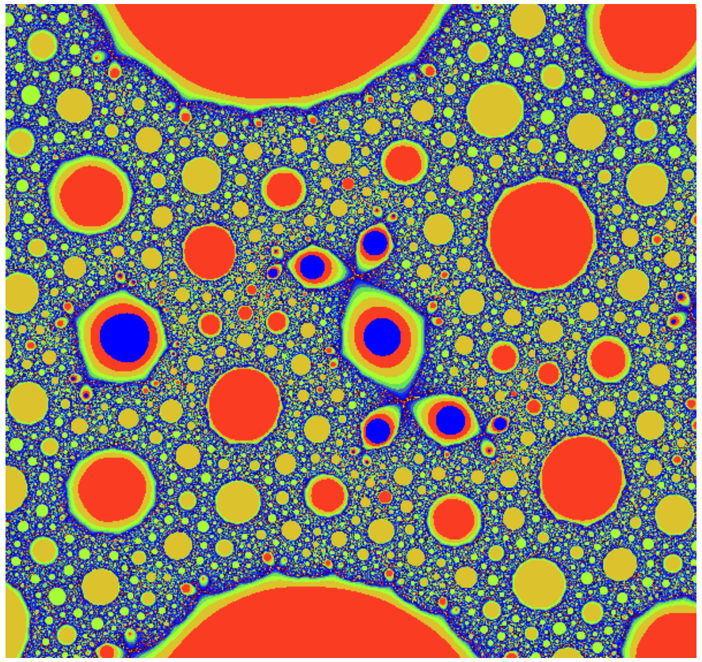
Questions:

1. Which connectivities can be achieved for a given family?
2. How to relate different perturbations through quasiconformal surgery
3. Transcendental perturbations of rational maps?

5.3 -- Invariant objects in rational iteration

Aspenberg + Canela + Fagella + Garijo + Gardini + Jarque

- **Matings** of rational maps:
  Sierpinski carpet + polynomial?



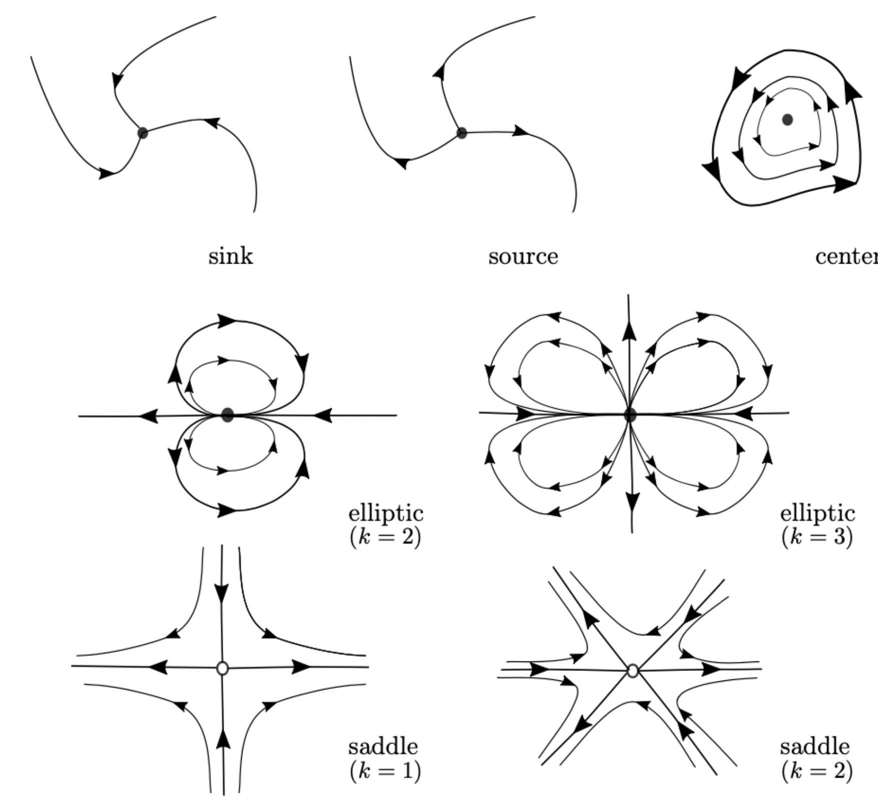
- Rational maps with denominators

- Effects of the Gaus-Seidel method to global dynamics.

6.3 -- The separatrix graph of a rational vector field
6.4 -- Bifurcations in rational vector fields

Garijo + Dias + Villadelprat

- **(complex) rational vector fields** are special types of planar vector fields



- Can be classified by their separatrix graph

Questions:
1. Can every graph be realizable?
2. Are there invariants that allow us to study bifurcations?

- Previous work for polynomial vector fields

- Construction of abstract Riemann surfaces using canonical regions.

Thank you for your attention!!