Abstract

My thesis concerns a particular model of a disordered chain of particles originating from a class of DNA chain models (A. Ponno, draft). The model equations of motion are

$$\iota\dot{\psi}_m = \sigma_m \psi_m + \varepsilon \left(\psi_{m+1} + \psi_{m-1} + |\psi_m|^2 \psi_m \right) \tag{1}$$

where the σ_m are random coefficients taking values ± 1 according to some probability distribution $\mathcal{P}$, ε is a small parameter and periodic boundary conditions are assumed ($m \in \mathbb{Z}_L$).
Equations (1) are the Hamilton equations associated to the Hamiltonian $H = h + \varepsilon P$ where

$$h = \sum_{m \in \mathbb{Z}_L} \sigma_m |\psi_m|^2$$

and

$$P = \sum_{m \in \mathbb{Z}_L} \psi_m^* \left(\psi_{m+1} + \psi_{m-1} \right) + \frac{1}{2} |\psi_m|^4$$

This model comes from the classical Peyard-Bishop model of DNA, and it is obtained taking into account the inhomogenety due to the distribution of different base pairs along the molecule. Under certain hypotesys, and making use of standard techniques of Hamiltonian perturbation theory, we show how such model reduce to a weakly dispersive, weakly non linear, discrete Schroedinger equation with "spin disorder".

We have studied the dynamics of the model above from the point of view of Hamiltonian perturbation theory, writing the Hamiltonian H of the model in normal form with respect to h up to order n. We show that at each order n the "truncated" (neglecting the remainder $\mathcal{R}_{n+1}$) normal form Hamiltonian

$$H^{(n)} = h + \sum_{k=1}^{n} \varepsilon^k P_k$$

consists in the sum of independent non-interacting Hamiltonians $\{\mathcal{H}_{\mathcal{D}_n^i}\}_{i \in I_n}$ defined each on certain disjoint domains $\mathcal{D}_n := \{\mathcal{D}_n^i\}_{i \in I_n}$ whose complexity increases with n. These domains are unions of blocks of nearby particles having the same value of the spin σ_m, and are such that at order n two blocks belong to the same domain if they are separated by at most $n-1$ consecutive spins of opposite sign with respect to that of the blocks.
So, each normalized perturbation term P_k describes how and how far the blocks interact between themselves, and the truncated Hamiltonian $H^{(n)}$ reads

$$H^{(n)} - \mathcal{R}_{n+1} = \sum_{\mathcal{D}_n} \mathcal{H}_{\mathcal{D}_n^i}$$

To prove the existence of this normal form $H^{(n)}$ of the Hamiltonian H, we have estimated each normal expansion term P_k and the remainder $\mathcal{R}_{n+1}$ of the expansion series.

We have obtained a Nekhoroshev-like result, determining an optimal truncation order $N \in \mathbb{N}$ of the perturbative expansion and an exponential estimate for the time of energy localization in the domains.

From the explicit expression of the normalized perturbation term P_k, it was possible to perform a phenomenological analysis of the model. We have extracted a complete description of the chaotic motion inside each single local domain of the family $\mathcal{D}_n$, explaining the role of the weak non linearity in the model and understanding that its presence does not change the optimal truncation order N and exponential time estimate of the stability of the localization in domain $\mathcal{D}_n$.

The last part we are facing now concerns the characterization of the localization domains in terms of the probability distribution $\mathcal{P}$ of the "spin disorder", and how its choise provides existence conditions of the localization phenomena, in particular on the perturbation coefficient ε.