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STATION KEEPING OF A SOLAR SAIL IN THE SOLAR SYSTEM

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Abstract

Solar sails are a concept of spacecraft propulsion where one takes advantage of the solar radiation pressure to propel a spacecraft. A first approach to model the dynamics of a solar sail in the Earth - Sun system is to consider the Circular Restricted Three Body Problem (CRTBP) adding the solar radiation pressure. In this framework, the effect of the solar sail creates a family of "artificial" equilibria parametrised by the orientation of the sail.

These equilibria offer interesting mission applications such as GeoStorm Warning Mission or Polar Observer, where a solar sail must remain close to a fixed location. In previous works[4,5] we have derived station keeping strategies for a solar sail close to equilibria using information on the dynamics of the systems. The main idea is to obtain the dynamical properties of the phase space close to an equilibrium point for a fixed sail orientation, and understand how these properties vary when we change the sail orientation. Then we can find a sequence of changes in the sail orientation so that the system acts in our favour, managing to maintain the trajectory of the sail close to the equilibrium point.

In this paper we study the performance of these strategies for a more realistic model, including the gravitational effect of other planets. More concretely, we will discuss how to extend these ideas when we consider that Sun - Earth move around their centre of mass in an elliptic way and include the gravitational attraction of Jupiter. This is the first step of a more challenging project where we want to check the robustness of the algorithms in a realistic setting.

1 Introduction

One of the main advantages of solar sails is that they open a wide range of challenging mission applications that cannot be achieved by a traditional spacecraft. Robert L. Forward in 1990 proposed to use a solar sail to hover one of the Earth's poles.[9] He would place a solar sail high above the ecliptic plane in such a way that the solar radiation pressure would counteract the Earth's gravitational attraction. He called it "Statite": the spacecraft that does not move. Nowadays, these ideas are being consider in the proposed Polar Observer and PolarSitter missions.[2,16] These mission concepts would enable to have constant monitoring of the Polar regions of the Earth for climatological studies.

Another interesting proposal is the so called GeoStorm mission.[16,21] Here the idea is to place a solar sail at an equilibrium point closer to the Sun than the Lagrangian point L_1 and displaced about 5° from the Earth-Sun line, enabling observations of the Sun's magnetic field having a constant communication with the Earth. This would enable to alert of Geomagnetic storms, doubling the actual alert time of the ACE[13] spacecraft (that is now orbiting on a Halo orbit around L_1).

All these missions require to maintain a solar sail in a fixed location. Nevertheless, most of these equilibria are unstable, hence a station keeping strategy is needed to maintain a solar sail close to equilibria for a long time.

In previous works[3,4,8] we used dynamical systems tools to develop a station keeping strategy for this situation in the Circular and Elliptical RTBP model. The key point was to know the position of the stable and unstable manifolds for a fixed sail orientation, and see now this one varied when the sail orientation is changed. This information was used to derive a sequence of changes on the sail orientation that kept the trajectory of the sail close to equilibria. We have already tested these algorithms with the GeoStorm and Polar Observer missions.[3,4,8] During our simulations we considered the RTBP with the effect of the solar radiation pressure as a model. We also included random errors on the position and velocity determination as well as on the sail orientation to test the robustness of these algorithms. We found that the most relevant sources of errors (the ones with more impact on the dynamics) are the errors on the sail orientation.

Now we want to test the robustness of these strategies when other perturbations are included into the system. To have a more realistic model one should include at least the gravitational attraction of the main bodies in the solar system. Another improvement can be introduced by considering a more realistic approximation to the sail performance, taking into account its shape and intrinsic properties.

In this paper we have included the gravitational effect of Jupiter, the most massive body in the solar system after the Sun. We will discuss how to extend the ideas of our previous works[3,4,8] when we consider a more complex model.

We have organised this paper as follows: In Section 2 we introduce the model used for the forces acting on the sail and explain the dynamical model for the motion of the sail. In Section 3 we will describe the main tools to derive these strategies in a more realistic model. We will first discuss how to find an appropriate nominal orbit (section 3.1) and how to find an appropriate reference system along the orbit that will help us to give an easy description of the systems dynamics (section 3.2). Then we will see how to put all these things together to derive a station keeping strategy (section 3.3). Finally, in Section 4 we will test the performance of this strategy on the GeoStorm mission.

2 Equations of Motion

2.1 The Solar Sail

The acceleration given by the sail depends on its orientation and its efficiency. As a first approach, one can consider that the force due to the radiation pressure is produced by the reflection of the photons emitted by the Sun on the surface of the sail.[17] For a more realistic model, one should include the force produced by the absorption of photons by the sail.[1,19] The force produced by reflection is directed along the normal direction to the surface of the sail, while the absorption is strictly in the opposite direction of the Sun. This means that the direction of the resultant force should be tilted from the normal direction to the surface of the sail.

In this paper we consider the simplest model for the sail, that is, a flat and perfectly reflecting sail. The force produced by the sail is,[17]

$$\vec{F}_{sail} = \beta \frac{G\ m_s}{r_{ps}^2} \langle \vec{r}_s, \vec{n} \rangle^2 \vec{n}, \tag{1}$$

where β is a constant defined as the sail lightness number, that accounts for the efficiency of the sail, G is the universal gravitational constant, m_s is the mass of the Sun and r_{ps} is the Sun - sail distance. The vectors $\vec{r}_s$ and $\vec{n}$ are unit vectors representing the Sun - sail direction and the normal direction to the surface of the sail respectively.

The sail orientation is parametrised by two angles, α and δ, which can be defined in many ways.[15,17,20] Here we define them as follows: (i) α is the angle between the projection of the Sun-sail line, $\vec{r}_s$, and the normal vector to the sail, $\vec{n}$, on the ecliptic plane; (ii) δ is the angle between the projection of the Sun - sail line, $\vec{r}_s$, and the normal vector to the sail, $\vec{n}$, on the $y = 0$ plane (see Fig. 1). These angles are related to the horizontal (α) and vertical (δ) displacement of the normal direction, $\vec{n}$, with respect to the Sun - line, r_s, in a given reference system.

If we consider (x_s, y_s, z_s) to be the position of the sail and (x_0, y_0, z_0) the one of the Sun, then it is obvious that $\vec{r}_s = (x_s - x_0, y_s - y_0, z_s - z_0)/r_{0s}$. Take spherical coordinates we have that $r_s = (\cos\phi(x, y)\cos\psi(x, y, z), \sin\phi(x, y)\cos\psi(x, y, z),$

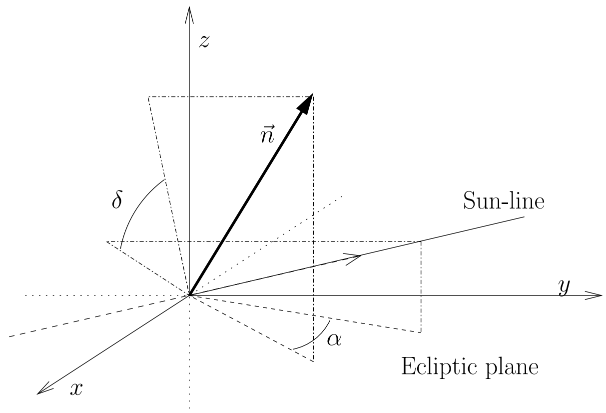

Figure 1: **Graphic representation of the two angles (α, δ) that define the sail orientation**

$\sin\psi(x,y,z))$, where

$$\phi(x,y) = \arctan\left(\frac{y_s - y_0}{x_s - x_0}\right),$$
$$\psi(x,y,z) = \arctan\left(\frac{z_s - z_0}{\sqrt{(x_s - x_0)^2 + (y_s - y_0)^2}}\right).$$

Notice that following the definitions given for α and δ we have that $\vec{n} = (n_x, n_y, n_z)$ can be expressed as:

$$\begin{aligned} n_x &= \cos(\phi(x,y) + \alpha)\cos(\psi(x,y,z) + \delta), \\ n_y &= \sin(\phi(x,y) + \alpha)\cos(\psi(x,y,z) + \delta), \\ n_z &= \sin(\psi(x,y,z) + \delta). \end{aligned}$$

2.2 N - Body Problem + Solar Sail

A solar sail in space is affected by the gravitational attraction of the main bodies. Hence, according to Newton's laws, the acceleration of the sail in the solar system will be given by:

$$\frac{d^2}{dt^2}\vec{r}_{sail} = G\sum_{i=0}^{9} m_i \frac{\vec{r}_i - \vec{r}_{sail}}{||\vec{r}_i - \vec{r}_{sail}||^3} + \vec{F}_{sail}, \qquad (2)$$

where $G = 6.67428 \cdot 10^{-11}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$ is the Universal gravitational constant, m_i is the mass of each of the bodies, $\vec{r}_i$ is their position.

We can consider the planets to be ordered by their distance to the Sun, where $i = 0$ corresponds to the Sun and $i = 1, \ldots, 9$ to the planets from Mercury to Neptune and the Moon. All the bodies from the Solar system will then evolve following their mutual gravitational attraction. If one wishes to have a more realistic model, their positions and velocities along time can be taken from the JPL ephemerides.

2.3 Sun - Earth - Jupiter - Sail model

If a solar sail is in the vicinity of the Earth, a good first approximation, is to considering only the gravitational effects of the Earth and Sun as well as the solar radiation pressure. If we fix the two bodies to evolve in a circular way around their mutual centre of mass, we have the Restricted Three Body Problem with Solar Sail (RTBPS), and has served us as a model in the past.

In this paper we will consider a more realistic model, which includes the gravitational effect of Jupiter and the eccentricity of the Earth's orbit. Jupiter is by far the largest planet in the solar system, with a mass more than twice the mass of all the other planet combined together. Hence, it is the most important perturbations to the trajectory of the sail.

The equations of motion are,

$$\begin{aligned} \ddot{x}_s &= \sum_{i=0}^{2} Gm_i \frac{x_i - x_s}{r_{is}^3} + \beta\frac{Gm_0}{r_{0s}^2}\langle\vec{r}_s, \vec{n}\rangle^2 n_x, \\ \ddot{y}_s &= \sum_{i=0}^{2} Gm_i \frac{y_i - y_s}{r_{is}^3} + \beta\frac{Gm_0}{r_{0s}^2}\langle\vec{r}_s, \vec{n}\rangle^2 n_y, \qquad (3) \\ \ddot{z}_s &= \sum_{i=0}^{2} Gm_i \frac{z_i - z_s}{r_{is}^3} + \beta\frac{Gm_0}{r_{0s}^2}\langle\vec{r}_s, \vec{n}\rangle^2 n_z, \end{aligned}$$

where (x_s, y_s, z_s) and (x_i, y_i, z_i) are the position of the solar sail and the planets respectively, where $i = 0, 1, 2$ stands for the Sun, Earth and Jupiter respectively. The constants Gm_i are their normalised masses and $r_{is} = \sqrt{(x_i - x_s)^2 + (y_i - y_s)^2 + (z_i - z_s)^2}$ are the planet - sail distances. Finally, $\vec{n} = (n_x, n_y, n_z)$ is the normal direction to the surface of the sail, and $\vec{r}_s = \vec{r}_{0s}/||r_{0s}||$ is the normalised Sun - sail direction.

Sun, Earth and Jupiter will evolve following their mutual gravitational interaction. Hence,

$$\begin{aligned} \ddot{x}_i &= \sum_{j=0}^{i-1} Gm_j \frac{x_i - x_j}{r_{ij}^3} + \sum_{j=i+1}^{2} Gm_j \frac{x_j - x_i}{r_{ij}^3}, \\ \ddot{y}_i &= \sum_{j=0}^{i-1} Gm_j \frac{y_i - y_j}{r_{ij}^3} + \sum_{j=i+1}^{2} Gm_j \frac{y_j - y_i}{r_{ij}^3}, \qquad (4) \\ \ddot{z}_i &= \sum_{j=0}^{i-1} Gm_j \frac{z_i - z_j}{r_{ij}^3} + \sum_{j=i+1}^{2} Gm_j \frac{z_j - z_i}{r_{ij}^3}, \end{aligned}$$

for $i = 0, 1, 2$.

The initial conditions for these bodies have been taken so that initially: the Sun - Earth couple orbits around their mutual centre of mass in an elliptic way with

$e = 0.0167$ and Jupiter orbits around this centre of mass on the same orbital plane in a circular way in the same orbital plane. This is a close first approximation to the real motion of these planets. So,

$$\begin{array}{rcllll}
(x_0, y_0, z_0) & = & (& m_1(1-e^2), & 0, & 0 \;), \\
(\dot{x}_0, \dot{y}_0, \dot{z}_0) & = & (& 0, & m_1\sqrt{\frac{1+e}{1-e}}, & 0 \;), \\
(x_1, y_1, z_1) & = & (& -m_0(1-e^2), & 0, & 0 \;), \\
(\dot{x}_1, \dot{y}_1, \dot{z}_1) & = & (& 0, & -m_0\sqrt{\frac{1+e}{1-e}}, & 0 \;), \\
(x_2, y_2, z_2) & = & (& -r_J\cos(\phi_0), & r_J\sin(\phi_0), & 0 \;), \\
(\dot{x}_2, \dot{y}_2, \dot{z}_2) & = & (& r_J\omega_J\sin(\phi_0), & r_J\omega_J\cos(\phi_0), & 0 \;),
\end{array}$$

where $r_J = 5.2$ AU, $\omega_J = 2\pi/11.86$ and $\phi_0 = \pi/6$.

We will use an inertial reference system. To be able to relate this model with the RTBPS, we have normalised the units of distance, mass and time so that: the Sun-Earth distance is 1, the mass of the Sun - Earth system is also 1 (i.e. $Gm_0 + Gm_1 = 1$), and their orbital period is 2π (i.e. $2\pi = 1$ year). Hence,

$$\begin{array}{rcl}
Gm_0 & = & 9.999969965194 \cdot 10^{-1}, \\
Gm_1 & = & 3.003480575402 \cdot 10^{-6}, \\
Gm_2 & = & 9.547890707253 \cdot 10^{-4}.
\end{array}$$

3 Station Keeping Strategy

As it has been discussed in previous works by McInnes et. al,[16,18] the artificial equilibria that appear on the CRTBP when the solar radiation pressure is added are in an interesting location for practical mission applications, such as the GeoStorm or the Polar Observer. Nevertheless, they are unstable and a station keeping strategy must be used to maintain the sail close to equilibria.

In previous papers,[4,5,7] we already discussed how to derive station keeping strategies around unstable equilibria the Circular RTBP using dynamical system tools. We also tested them and discussed their robustness when different sources of errors were included in the simulations (both on the position and velocity determination and the sail orientation).

Now we want to check the robustness of these strategies when other perturbations are added to the system, such as the fact that the two primaries (Sun and Earth) actually orbit around their centre of mass in an elliptic way, and the gravitational attraction of Jupiter, the largest body in the Solar system after the Sun.

It is clear that when we include these perturbing effects to RTBPS the artificial equilibria no longer exist, but if the perturbations are small enough, there still exist natural trajectories that remain close to them and that share the same linear behaviour.[14] In Section 3.1 we will discuss how to compute these nominal orbits. Our goal will be to remain close to these natural trajectories.

We recall that the key point of the strategies used in our work[4,5] was to understand the geometry of the phase space and how it is affected by variations on the sail orientation. In Section 3.2 we will describe the linear dynamics around these objects and how to find an appropriate reference system.

The main idea is to keep the sail orientation fixed for a certain time, letting the dynamics of the system act. When the sail is escaping from the nominal orbit we chose a new sail orientation that brings these trajectory back. In Section 3.3 we will see how to find this appropriate new sail orientation and how to put all these ideas together to derive the station keeping algorithms.

We must mention that some of the ideas behind our approach are based on the previous works by Gomez et al.[10,12] on the station keeping around Halo orbits with a "traditional" thruster.

3.1 Nominal Orbit

Our goal is to maintain the trajectory of a solar sail close to one of the artificial equilibrium point that appear in the Circular RTBP when the solar radiation pressure is included. As discussed in,[8] if we consider the Elliptic RTBPS (i.e. for the moment we neglect Jupiter) we no longer have artificial equilibria, these ones have been replaced by 2π-periodic orbits. This is because the Elliptic RTBPS can be seen as a 2π-periodic perturbation of the Circular RTBP. Nevertheless, these periodic orbits remain close to the equilibrium point in the Circular RTBP and share the same qualitative behaviour. In our previous paper[8] we used them as nominal orbits for station keeping.

When we include other perturbations to the system, for example, the gravitational effect of Jupiter in our model. The system is no longer a periodic perturbation of the Circular RTBP, hence these periodic orbits will no longer exist. Nevertheless, there still exist natural trajectories of the system that remain close to these ones.[14] We will use them as nominal orbits for our station keeping. Moreover, the qualitative behaviour around these orbits is similar to the behaviour around the 2π-periodic in the Elliptic RTBP. Hence the ideas behind our station keeping strategies still apply.

To find a good nominal orbit we have implemented a parallel shooting method[11] to get a solution in the Sun - Earth - Jupiter model very similar to the one in the Elliptic RTBP Sun - Earth model.

Let us give some detail on it. First we split the time span $[0, T_{end}]$ in which we want to find the nominal orbit into several pieces $[t_i, t_{i+1}]$, $i = 0, \dots n-1$, verifying $t_0 = 0$, $t_n = T_{end}$ and $\tau = t_{i+1} - t_i = T_{end}/n$. In our examples we have considered $T_{end} = 20$ years (the maximum duration of our mission) and $\tau = 0.5$ years (half the period of the periodic orbits in the ERTBP), hence $n = 40$.

If x_i for $i = 0, \dots, n$ are $n+1$ points in the phase space and we define $\phi_\tau(t_i, x_i)$ as the solution given by the initial condition (t_i, x_i) at time $t_{i+1} = t_i + \tau$. These points will belong to the nominal orbit if they satisfy,

$$\phi_\tau(t_i, x_i) = x_{i+1} \quad \text{for } i = 0, \dots, n-1.$$

This leads to having to solve a non-linear equation with $6n$ equations and $6n+6$ unknowns. To do this, we have used a Newton method taking as initial guess the values provided by the periodic orbit in the Elliptic RTBP. Due to the fact that we have more unknowns than equations we have added six more conditions: we have fixed the initial positions (the first three components of x_0) and the final ones (the first three components of x_n).

3.2 Linear Dynamics

The study of the linear dynamics is very similar to the done by Gomez et al.[10,12] where they described the linear dynamics close to Halo orbits in the RTBP. Here we will discuss the main details.

The first step to study the behaviour close to the nominal orbit is done throughout the first order variational equations. If we write the equations of motion in Eq. 3 as $\dot{x} = F(x, t)$, then the first variational equations are

$$\dot{A} = DF(x(t), t)A, \quad A \in \mathcal{L}(\mathbb{R}^6, \mathbb{R}^6), \tag{5}$$

with the initial condition $A(0) = Id$.

If ϕ denotes the flow associated to Eq. (3) and $\phi_\tau(t_0, x_0)$ the image of the point $x_0 \in \mathbb{R}^6$ after τ units of time. The solution $A(\tau)$ of Eq. (5) is the differential flow, $D\phi_\tau(t_0, x_0)$, of $\phi_\tau(t_0, x_0)$ with respect to the initial condition x_0. For $h \in \mathbb{R}^6$ (with $\|h\|$ small enough), we have

$$\phi_\tau(t_0, x_0 + h) = \phi_\tau(t_0, x_0) + D\phi_\tau(t_0, x_0) \cdot h + O(|h|^2).$$

Therefore, $\phi_\tau(t_0, x_0) + A(\tau) \cdot h$ gives a good approximation of $\phi_\tau(t_0, x_0 + h)$ provided that h is small.

The variational flow of the nominal orbit, $A(\tau)$, gives information on the dynamics close to it. The linear behaviour around this piece of the orbit will be determined by the matrix $M = A(T_{end})$.

To avoid problems in the integration due to the high instability of the system, we have split the nominal orbit into N pieces. Each piece corresponds to one revolution of the Earth around the Sun, this is way from now on we will refer to each piece as revolution of the nominal orbit. Associated to each revolution we have the variational matrix A_k in normalised coordinates. It is easy to check that $M = A_N \times A_{N-1} \times \cdots \times A_1$.

Due to the large values of the unstable eigenvalues of each one of the matrices A_k (roughly 396) it is not possible to perform a direct computation of the eigenvalues of M because of the possible overflow during the computation of M. We must take into account that the dominant eigenvalue of M is of the order of 396^N. There exist procedures[10] that can be done to deal with this problem and find all the eigenvalues and eigenvectors of M.

In our case, we will use each of the individual matrices A_k and find their eigenvectors. In this way we can describe the local local dynamics for each revolution. It is true that this implies a certain discontinuity every time we change from one revolution to another, but it has given good results in the past.[12]

We can consider each revolution to be 1 year and that the linear dynamics of the nominal orbit during n the kth revolution is described by A_k. We note that there is not many differences in the qualitative and quantitative dynamics after 1 revolution. Hence, this is a good first approach for the global dynamics. We can understand the dynamics at each revolution by finding the eigenvalues and eigenvectors of A_k.

For each revolution, the eigenvalues $(\lambda_{1,\dots,6})$ of the A_k, are very similar and are as follows: $\lambda_{1,2}$ are real with $\lambda_1 > 1, \lambda_2 < 1$, the others $\lambda_{3,4,5,6}$ are complex pairs of conjugate eigenvalues, $\lambda_3 = \bar{\lambda}_4$ and $\lambda_5 = \bar{\lambda}_6$.

The previous three pairs of eigenvalues have the following geometrical meaning:

- The first pair (λ_1, λ_2) are related to the (strong) hyperbolic character of the orbit. The value λ_1 is the largest in absolute value, and is related to the eigenvector $e_1(0)$, which gives the most expanding direction. Using $D\phi_\tau$ we can get the image of this vector under the variational flow: $e_1(\tau) = D\phi_\tau e_1(0)$. At each point of the orbit, the vector $e_1(\tau)$ together with the vector tangent to the orbit, span a plane that is tangent to the local unstable manifold (W^u_{loc}). In the same way λ_2 and its related eigenvector $e_2(0)$ are related to the stable manifold and $e_2(\tau) = D\phi_\tau e_2(0)$.

- The other two couples $(\lambda_3, \lambda_4 = \bar{\lambda}_3)$ and $(\lambda_5, \lambda_6 = \bar{\lambda}_5)$ are complex conjugate and their modulus is close to 1. The matrix M, restricted to the plane spanned by the real and imaginary parts of the eigenvectors associated to λ_3, λ_4 (and λ_5, λ_6) is a rotation with a small dissipation or expansion, so that the trajectories on these planes spiral inwards or outwards. A_k restricted to these planes has the form,

$$\begin{pmatrix} \Delta_i \cos\Gamma_i & -\Delta_i \sin\Gamma_i \\ \Delta_i \sin\Gamma_i & \Delta_i \cos\Gamma_i \end{pmatrix},$$

  where $\Delta_{1,2}$ denotes the modulus of λ_3 and λ_5 respectively, and are the rate of contraction. $\Gamma_{1,2}$ denotes the argument of λ_3 and λ_5 respectively, and they account for the rotation rate around the orbit.

- We have that $|\lambda_{3,4,5,6}| \ll |\lambda_1|$, hence the most expanding direction (by far) is given by $e_1(\tau)$.

To sum up, in a suitable basis the variational flow, A_k, associated to the nominal orbit can be written as, $B_k =$

$$\left(\begin{array}{cc|cc|cc}
\lambda_1^k & & & & & \\
 & \lambda_2^k & & & 0 & \\
\hline
 & & \Delta_1^k \cos\Gamma_1^k & -\Delta_1^k \sin\Gamma_1^k & & \\
 & & \Delta_1^k \sin\Gamma_1^k & \Delta_1^k \cos\Gamma_1^k & & \\
\hline
 & 0 & & & \Delta_2^k \cos\Gamma_2^k & -\Delta_2^k \sin\Gamma_2^k \\
 & & & & \Delta_2^k \sin\Gamma_2^k & \Delta_2^k \cos\Gamma_2^k
\end{array}\right).$$

The functions $e_i(\tau) = D\phi_\tau \cdot e_i(0)$, $i = 1, \ldots, 6$, give an idea of the variation of the phase space properties in a small neighbourhood of the periodic orbit. We will use a modification of them, the so called the Floquet modes[6,8,12] $\bar{e}_i(\tau)$ to track a trajectory close to the nominal orbit and give a simple description of its dynamics.

3.2.1 The Floquet modes

We recall that, without the effect of Jupiter, the nominal orbit would be a periodic orbit with a period of 1 year. Due to the presence of Jupiter, this orbit is no longer periodic, but it almost closes after one year. For this reason, we will use ideas from the classical Floquet theory to represent the linear dynamics around it.

The Floquet modes provide us a local reference system for each revolution of the orbit, which is very useful to track the relative position between the spacecraft trajectory invariant manifolds of the nominal orbit.

The Floquet modes $\bar{e}_i(\tau)$ $(i = 1, \ldots, 6)$ are six 2π-periodic time-dependent vectors such that, if we call $P(t)$ to the matrix that has the vectors $\bar{e}_i(\tau)$ as columns, then the change of variables $x = P(t)z$, takes the linearised equation around the 2π-slice of the nominal orbit, $\dot{x} = A_k(\tau)x$, to an equation with constant coefficients $\dot{z} = B_k z$.

One of the main advantages of using the Floquet bases, is the fact that it is periodic. The after one revolution, when we change from one piece of orbit to another, there will be a small discontinuity between the two reference systems. This essentially translates on a small jump in the phase space made by the sail every revolution.

Following[12] we define the first and second Floquet mode taking into account that the rate of escape and approach, after one revolution, along the unstable and stable manifolds is exponential:

$$\begin{aligned}
\bar{e}_1(t) &= e_1(\tau)\exp\left(-\tfrac{\tau}{T}\ln\lambda_1\right), \\
\bar{e}_2(t) &= e_2(\tau)\exp\left(-\tfrac{\tau}{T}\ln\lambda_2\right).
\end{aligned}$$

The other pairs are computed by taking into account that, after one revolution, the plane generated by the real and imaginary parts of the eigenvectors associated to (λ_3, λ_4) and (λ_5, λ_6) is a rotation of angle $\Gamma_{1,2}$ and a dissipation/expansion by a factor of $\Delta_{1,2}$:

$$\begin{aligned}
\bar{e}_3(t) &= [\cos\left(-\Gamma_1 \tfrac{\tau}{T}\right) e_3(\tau) - \sin\left(-\Gamma_1 \tfrac{\tau}{T}\right) e_4(\tau)]\epsilon_1,\\
\bar{e}_4(t) &= [\sin\left(-\Gamma_1 \tfrac{\tau}{T}\right) e_3(\tau) + \cos\left(-\Gamma_1 \tfrac{\tau}{T}\right) e_4(\tau)]\epsilon_1,\\
\bar{e}_5(t) &= [\cos\left(-\Gamma_2 \tfrac{\tau}{T}\right) e_5(\tau) - \sin\left(-\Gamma_2 \tfrac{\tau}{T}\right) e_6(\tau)]\epsilon_2,\\
\bar{e}_6(t) &= [\sin\left(-\Gamma_2 \tfrac{\tau}{T}\right) e_5(\tau) + \cos\left(-\Gamma_2 \tfrac{\tau}{T}\right) e_6(\tau)]\epsilon_2.
\end{aligned}$$

Where $\epsilon_i = \exp(-\frac{\tau}{T}\ln\Delta_i)$, $\tau = t - k\cdot T$ is a normalised time so that all the Floquet modes are taken between $[0, 2\pi]$. Here k stands for the revolutions we are considering and $T = 2\pi$ the time it takes to do one revolution.

3.2.2 Reference System

To build our reference frame, we split the time interval of the mission duration $[0, T_{end}]$ into N revolutions, having N time intervals $I_i = [t_i, t_{i+1}]$, $i = 0, \ldots, N-1$, where $t_0 = 0$, $t_i = t_{i-1} + h$ and $h = T_{end}/N$. In all of our examples we have considered $T_{end} = 20$ years (the maximum duration of our mission) and $N = 20$ (1 revolution = 1 year).

For each time interval I_i we have computed the Floquet modes associated to the variational flow A_i and stored them so that we can easily be recompute. Now we define our reference system as:

$$\{\ N_0(t);\ \vec{v}_1(t), \vec{v}_2(t), \vec{v}_3(t), \vec{v}_4(t), \vec{v}_5(t), \vec{v}_6(t)\ \}, \qquad (6)$$

where $N_0(t)$ is position and velocities of the nominal orbit at time t, and $\vec{v}_{1,\ldots,6}(t)$ corresponds to the Floquet modes of the time interval I_i into which t falls, formally defined as:

$$\vec{v}_i(t) = \sum_{k=0}^{N} \chi(I_k)\bar{e}_i^k(t),$$

where $\chi(t) = \{1 \text{ if } t \in T_k,\ 1 \text{ if } t \notin T_k\}$ and $\bar{e}_i^k(t)$ is the Floquet mode associated to the kth revolution.

Notice that the directions in this reference frame are discontinuous at each revolution. This means that at each revolution there will be a small jump of the trajectory in the phase space. Nevertheless the difference between the different eigenvectors of A_k is very small and these jumps will be almost negligible.

Now the dynamics around the nominal orbit is quite simple. If $N_0(\tau)$ denotes the point on the nominal orbit at time τ, then $\vec{v}_1(\tau)$ is the direction of the unstable manifold. When this base point follows the nominal orbit, the vector $\vec{v}_1(\tau)$ moves along the orbit following the (two-dimensional) unstable manifold. In the same way, the vector $\vec{v}_2(\tau)$ follows the stable manifold along the orbit. Moreover, these two directions generate a plane that moves along the orbit, on which the dynamics is a saddle.

For each point of the nominal orbit, the couple $\vec{v}_3(\tau)$, $\vec{v}_4(\tau)$ spans a plane that is tangent to another invariant manifold of the orbit. This plane spans a three-dimensional manifold when the base point moves along the orbit. The dynamics on this manifold can be visualised as a spiral motion (towards the nominal orbit) on the plane $(\vec{v}_3(\tau), \vec{v}_4(\tau))$ at the same time that the plane moves along the orbit. In a similar way, the couple $\vec{v}_5(\tau)$, $\vec{v}_6(\tau)$ spans another three-dimensional manifold, on which the dynamics is again a spiral motion (but now escaping from the nominal orbit) composed with the motion along the orbit.

The growing (or compression) of these spiral motions is due to the real part of $\lambda_{3,4}$ and $\lambda_{5,6}$, which is nonzero but very small. For this reason the spiralling motion is very small (almost circular) and, to compute the control strategy, we will assume that this motions is not an spiral but a rotation. Of course, the simulations of the control strategy are done without this assumption, and the control is good enough to compensate for the spiralling components (similar ideas were used in[4]).

3.3 Control Strategy

We have just shown that using an appropriate reference system, the dynamics around the nominal orbit can be seen as the cross product of a saddle and two centres. This means that for a fixed sail orientation (α_0, δ_0) a trajectory that starts close to the nominal orbit will escape along the unstable directions while rotating around the centre projections.

If we change the sail orientation $(\alpha_0 + \Delta\alpha, \delta_0 + \Delta\delta)$, the qualitative phase space behaviour will be the same, but the position of the nominal orbit, stable and unstable manifolds that rule the dynamics will be shifted. Hence, the the trajectory will now escape along the new unstable invariant manifold.[3,4]

In order to control the sail's trajectory we want to find a new sail orientation such that the new unstable manifold will bring the trajectory close to the stable manifold of our nominal orbit. Once the trajectory is close to this stable manifold we will restore the sail orientation to (α_0, δ_0) and repeat this process over and over to control the instability due to the saddle during the mission lifetime. Nevertheless, we must also take into account the centre projection of the sail's trajectory. A sequence of changes in the sail orientation derive in a sequence of rotations around different nominal orbits, and this can result unbounded.

Now that we understand how the linear dynamics around the nominal orbit works, and how changes in the sail orientation affect the sail's trajectory, we can derive an efficient algorithm to maintain the sail close to the desired nominal orbit. The key point of this strategy is how to find the appropriate changes in the sail's orientation so the phase space acts in our favour. For this we will use the first order variational equations w.r.t. the sail orientation.

3.3.1 First order variational flow

The first order variational flow gives us the information on how small variations in the initial conditions affect the final trajectory. In the same way the first order variational w.r.t. the two angles gives us the information on how small variations on the sail orientation will affect the final trajectory.

Let $\phi_h(t_0, x_0, \alpha_0, \delta_0)$ be the flow at time $t_1 = t_0 + h$ of our vector field starting at time t_0 for $(x_0, \alpha_0, \delta_0)$, then

$$\begin{aligned}\phi_h(t_0, x_0, \alpha_0 + \Delta\alpha, \delta_0 + \Delta\delta) = \phi_h(t_0, x_0, \alpha_0, \delta_0) + \\ \frac{\partial \phi_h}{\partial \alpha}(t_0, x_0, \alpha_0, \delta_0) \cdot \Delta\alpha + \frac{\partial \phi_h}{\partial \delta}(t_0, x_0, \alpha_0, \delta_0) \cdot \Delta\delta,\end{aligned} \tag{7}$$

is a first order approximation of the final state if a change $\Delta\alpha, \Delta\delta$ is made at time $t = t_0$. With this we have an explicit function of the final states of the trajectory as a function of the two angles.

3.3.2 The algorithm

We start with a solar sail close to the nominal orbit N_0 with a fixed sail orientation α_0, δ_0. We take the reference system $\{N_0(t); \vec{v}_i(t)\}_{i=1,\ldots,6}$, where $\vec{v}_1(t), \vec{v}_2(t)$ are the stable and unstable directions at each time and the other two pairs define the two centre planes.

We will use this reference system to track the trajectory and make decisions on when and how to change the sail orientation. To fix notation, if $\varphi(t_0)$ is the position and velocity of the solar sail at time t_0, then in this reference system,

$$\varphi(t_0) = N_0(t_0) + \sum_{i=1}^{6} s_i(t_0)\vec{v}_i(t_0).$$

For each mission we must define 3 parameters which will depend on the mission requirements and the dynamics of the system around the nominal orbit. These are: ε_{max}, the maximum distance to the stable direction allowed, used to decide when to change the sail orientation; dt_{min} and dt_{max} the minimum and maximum time between manoeuvres allowed.

We will proceed as follows, when we are close to $N_0(t_0)$ we set the sail orientation $\alpha = \alpha_0, \delta = \delta_0$. Due to the saddle, the trajectory will escape along the unstable direction. When $|s_1(t_1)| > \varepsilon_{max}$, we consider that the sail is about to escape and we need to change the sail orientation. We use the first order variational flow to find a new sail orientation α_1, δ_1 and time $dt_1 \in [dt_{min}, dt_{max}]$ so that by changing the sail orientation now, $t = t_1$, then at a certain time $t = t_1 + dt_1$ the sail trajectory will be close to the nominal orbit, $N_0(t_1 + dt_1)$. Finally, at $t_1 + dt_1$ we will restore the sail orientation to α_0, δ_0 and repeat these process during the mission lifetime.

3.3.3 Finding α_1, δ_1 and dt_1

Let us assume that we have $|s_1(t_1)| > \varepsilon_{max}$ at time $t = t_1$ and we need to chose a new sail orientation. Eq. 7 gives us a map of how a small change in the sail orientation $\Delta\alpha, \Delta\delta$ at $t = t_1$ will affect the sail trajectory at time $t = t_1 + h$.

We want to find $\Delta\alpha_1, \Delta\delta_1$ and dt_1 so that the flow at time $t = t_1 + dt_1$ is close to the stable manifold, $|s_1(t_1+dt_1)|$ small, and the centre projections, $(s_3(t_1+dt_1), s_4(t_1+dt_1))$ and $(s_5(t_1+dt_1), s_6(t_1+dt_1))$, do not grow .

We will proceed as follows:

1. Let us take $\tilde{t_i}$ for $i = 0, \ldots, n$ in the time interval $[t_1 + dt_{min}, t_1 + dt_{max}]$ where $\tilde{t_i} = t_1 + dt_{min} + i \cdot dt$ and $dt = (dt_{max} - dt_{min})/n$. For each $\tilde{t_i}$ we compute the variational map given by Eq. 7.
2. For each $\tilde{t_i}$ we find $\Delta\alpha_i, \Delta\delta_i$ such that, $s_1(\tilde{t_i}) = s_5(\tilde{t_i}) = s_6(\tilde{t_i}) = 0$. Notice that this reduces to solving a linear system with 2 unknowns and 3 equations, which we solve using the least square method.

At the end we have a set of $\{\tilde{t_i}, \Delta\alpha_i, \Delta\delta_i\}$ such that, $||(s_1(\tilde{t_i}), s_5(\tilde{t_i}), s_6(\tilde{t_i}))||$ is small.

3. From the set of $\{\tilde{t_i}, \Delta\alpha_i, \Delta\delta_i\}$ found in step 2 we choose the j so that $||(s_3(\tilde{t_j}), s_4(\tilde{t_j}))||$ is the smallest.

The desired parameters to bring the sail back to the nominal orbit are:

$$\alpha_1 = \alpha_0 + \Delta\alpha_j, \quad \delta_1 = \delta_0 + \Delta\delta_j, \quad dt_1 = \tilde{t_j} - t_1. \quad (8)$$

We must mention that all the strategies described here use information from the linear dynamics of the system to make decisions on the changes of the sail orientation. Nevertheless, the complete set of equations is taken into account during the simulations.

4 Mission Application

We have taken the GeoStorm Warning Mission[16,21] as an example to test our strategies. The main goal of the mission is to provide enhanced warning of geomagnetic storms to allow operation teams to take preventive actions to protect vulnerable systems. Currently predictions of future activity are made by the National Oceanic Atmospheric Administration (NOAA) Space Environment Centre in Colorado using terrestrial and real-time solar wind data obtained from the Advanced Compositions Explorer (ACE) spacecraft. The ACE spacecraft is stationed on a halo orbit near L_1, at about 0.01 AU from the Earth. From this position the spacecraft has continuous view of the Sun and communication with the Earth.

The enhanced storm warning provided by ACE is limited by the need to orbit the L_1 point and can only provide predictions of 1 hour in advance. However, since solar sails add an extra force to the dynamics of the orbit, the location of L_1 can be artificially displaced. The goal of GeoStorm is to station a solar sail twice as far from the Earth than L_1 while remaining close to the Sun - Earth line as can be seen in Fig. 2. This will enable us to at least double the time of alert that we have close to L_1.

If we want to displace a solar sail at a double distance from the classical Sun - Earth L_1 point, we need a sail lightness number $\beta = 0.051689$ (which corresponds to $a_0 \approx 0.3$ mm/s^2).[16,17,21] Moreover, as we need constant communication with the Earth, its position must

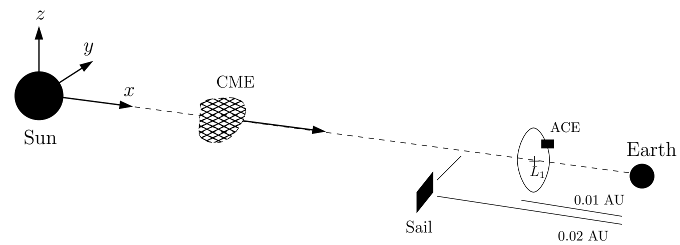

Figure 2: **Schematic representation of the position of the GeoStorm Mission (not to scale).**

be displaced approximately 5^o from the Sun - Earth line. The equilibrium point that we find within this constraints in the Circular RTBP is unstable, hence a station keeping strategy is required.

In our previous papers[4,8] we discussed the robustness of our strategy in the Circular and Elliptical RTBP for this mission, now we want to study its behaviour when we include the gravitational effect of the other planets. In this study only Jupiter is included.

4.1 Mission Parameters

Taking the RTBPS as a model, to find a fixed point that matches the GeoStorm Mission requirements we need: $\beta = 0.051689$, $\alpha_0 = 0.7897°$ and $\delta = 0°$.

We have the the periodic orbit replacing the fixed point in the Elliptic RTBP as an initial seed to find the nominal orbit for the model that also include the effect of Jupiter, and computed it over 20 years using the a parallel shooting method and its piecewise reference system, as explained in Sections 3.1 and 3.2.

Finally, as mission parameters, we have taken $\varepsilon_{max} = 5 \cdot 10^{-5}$AU (the escape distance), $dt_{min} = 2$ days and $dt_{max} = 169$ days (the minimum and maximum time between manoeuvres).

4.2 Mission Results

We have taken several random initial conditions close to the nominal orbit at $t = 0$, and applied to each of them the station keeping strategies up to 20 years. During each mission we have measured the average time between manoeuvre and the variation of the sail orientation (α, δ) along time.

In Figure 3 we see the variation on the two angles that define the sail orientation. We can see that the average variation of α is of $1°$ and δ remains almost fixed through time. The trajectory takes, in average, about 100 days to escape, and two quick manoeuvres every 2 days are done each time we want to bring back the trajectory. Notice that during the two days while we are bringing the sail back, the sail need to moves only about 1°, which seems reasonable.

In Figure 4 we can see the trajectory of a particular initial condition after applying the control strategy. This trajectory is being plotted using a rotating reference system that keeps the Sun - Earth line fixed. As we can see the trajectory remains close to equilibria.

In Figure 5 we show the same trajectory but in the reference system used by the station keeping to make decisions. Notice that the trajectory on the saddle projection is a sequence of saddle connections that remains bounded. While in the centre projections we have a sequence of rotations that remain bounded through time.

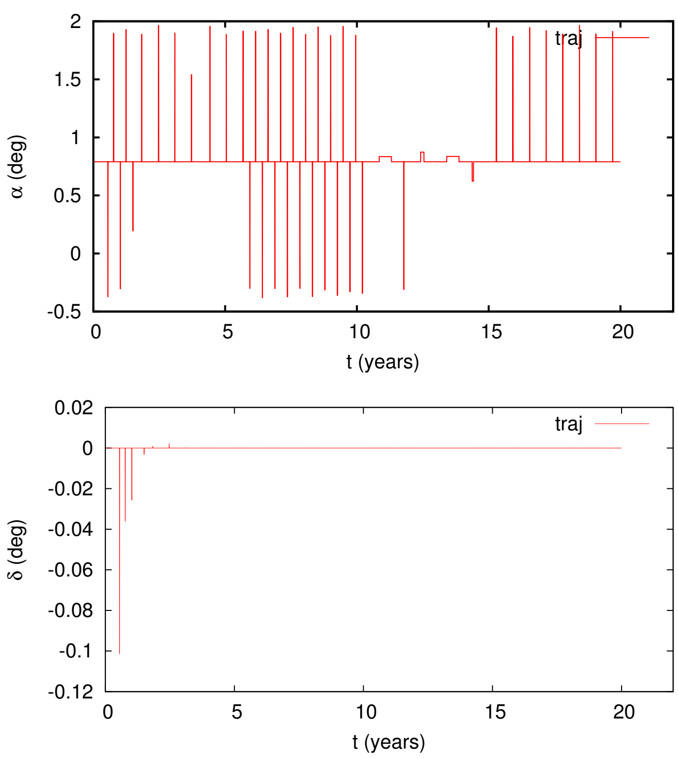

Figure 3: **Variation of the sail orientation α (top) and δ (bottom) along time.**

5 Conclusions and Future Work

In this work we have considered a solar sail near SL_1 point of the Sun - Earth system and describe a station

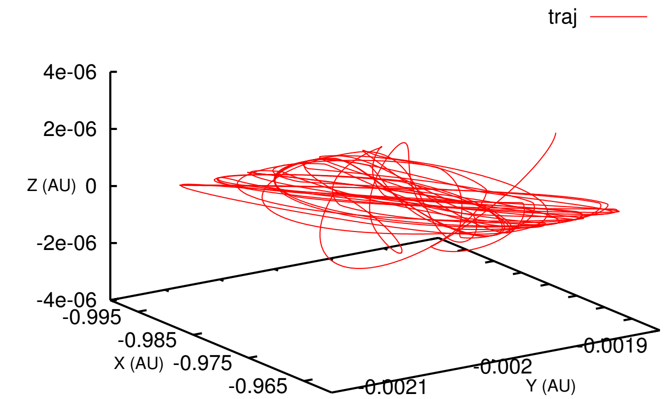

Figure 4: **Trajectory of the solar sail for 20 years in a rotating reference system so that the Sun - Earth line is fixed.**

keeping strategy to keep the trajectory of the solar sail close to it. As a concrete example, we have focused on the GeoStorm mission. The original part of this work is that, instead of using the RTBP as a model we have included the effect of the eccentricity of the Earth's orbit as well as the gravitational effect of Jupiter.

For the Sun-Earth-Jupiter model we show how to computed a nominal orbit that is close to the fixed point in the Circular RTBP and studied the local dynamics around it. Then we have used this information to derive a station keeping algorithm. The techniques used here are not based on classical control theory but on strongly rely on dynamical system tools. We already used these tools in the Circular RTBP[4] and the Elliptic RTBP.[8] Here we show how the same ideas extend naturally to a more complex model.

Finally, we have tested this algorithm for the particular case: the GeoStorm mission. Where we have managed to maintain the solar sail close to the nominal orbit over 20 years.

For a more complete study we still need to test its performance when different sources of errors that can occurs during a mission are taken into account. For example, errors on the position and velocity determination, as well as errors on the sail orientation. We have already tested these in the past when the RTBP with a solar sail was used as a model. Hence, we believe that these strategies should be robust enough.

Moreover, we are currently working on using these same ideas when gravitational perturbation of the complete Solar System is taken into account.

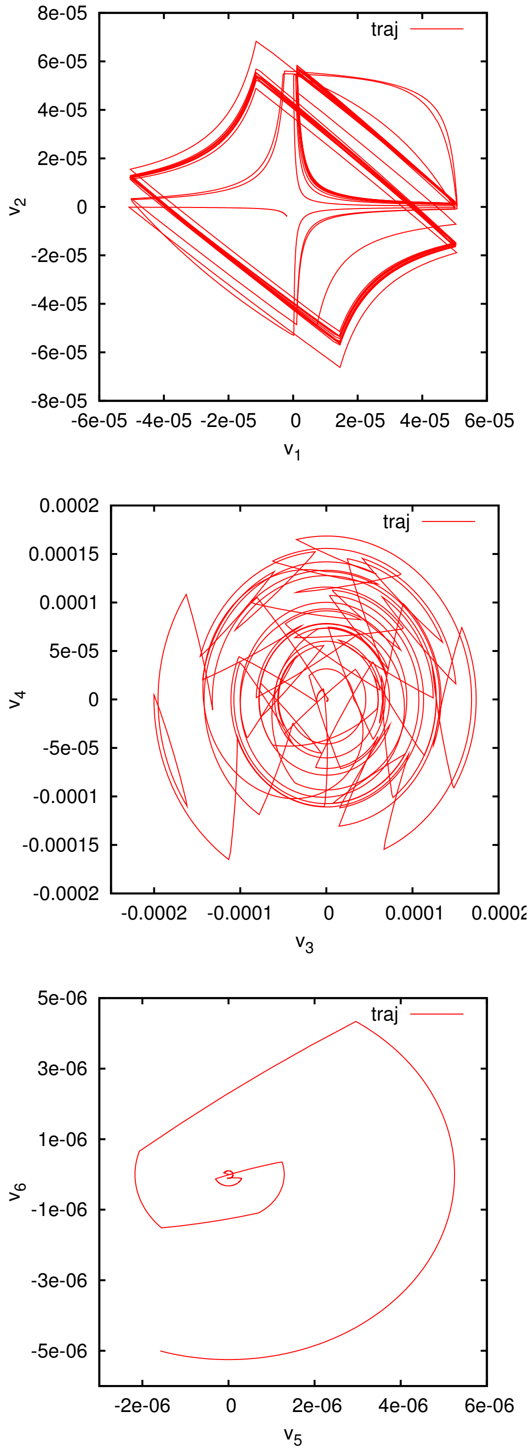

Figure 5: **Trajectory of the solar sail for 20 years. From top to bottom projections of this trajectory on the saddle projection generated by** $(\vec{v}_1(t), \vec{v}_2(t))$**, and the two other centre projections generated by** $(\vec{v}_3(t), \vec{v}_4(t))$ **and** $(\vec{v}_5(t), \vec{v}_6(t))$**.**

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