Control of satellite relative motion using low thrust time-delay feedback control

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Satellite formations has enabled a new kind of missions:

- A mission specific to formation flying is the continuous measurement of both surface reflectance and atmospheric optical thickness as is shown by H. Chepfer et al. [4] and J. Biggs and C. McInnes [5].
- Continuous remote sensing of large areas over Earth, M. King and et. al. [1].
- Earth radiance measurements at different wavelengths,
- Earth imaging in a broad spectral range,
- Magnetic field monitoring.
Satellite formations at Lagrange points (II)

In order to maintain a formation at one of the Lagrange points, one must achieve the following:

- Maintaining the formation with the desired relative motion,
- Find families of artificial orbits with the same orbit period, it was shown by J. Biggs [5], [6] that this families exist,
- Stability of the orbit, focused on the stability of halo orbits,
- Tracking a reference orbit,
- Relative motion control and stability (bounding the relative motion).
The circular restricted three body problem

- We consider the circular restricted three body problem with continues thrust.
- Initial natural halo orbits around L1 and L2, in the Earth Sun system, are determined using the Lindstedt-Poincaré method.
Families of artificial halo orbits (I)

- Families of artificial halo orbits are determined using numerical continuation.
- The setup for the continuation:

\[ t \to t/P \]

\[ \dot{X}(t) = P\dot{Y}(Y(t), t) \]

\[ P = \frac{L}{\int_0^1 \| f(Y(s)) \| ds} \]

\[ F(Y) = \begin{bmatrix}
\dot{X} - Pf(Y(t), t), \\
X_0 - Pf(Y(1), 1)
\end{bmatrix} 
\begin{bmatrix}
P - \frac{L}{\int_0^1 \| f(Y(s)) \| ds} \\
\end{bmatrix} \]

\[ F(Y_1) = 0 \]

\[ (Y_1 - Y_0)^T \dot{Y}_0 - \Delta s = 0 \]

\[ \| \dot{Y}_0 \| = 1 \]

\[ Y = (X(.), L, a_x, a_y) \]
Families of artificial halo orbits (II)

The L1 family:
Families of artificial halo orbits (III)

The L2 family:
Families of artificial halo orbits (III)

Orbit amplitude and thrust amplitude:
Stability of halo orbits study with Floquet theory

- The Floquet exponent:
  \[ \lambda_i = e^{\alpha_i T}. \]

- Form of the characteristic exponent:
  \[ \lambda_1 > 1, \lambda_2 < 1, \lambda_1 \cdot \lambda_2 = 1, \lambda_3 = \lambda_4 = 1 \text{ and } \lambda_5 = \lambda_6 \]

- Stability indices $K_1$ and $K_2$ [14]:
  \[ K_i = \frac{1}{Re(\lambda_i)} + Re(\lambda_i), \ i = 1, 2, \]
Stability of halo orbits (II)

Stability of halo orbits study with Floquet theory
Due to numerical error and of the high degree of instability, a small halo orbit will not close when using numerical integration.

A feedback mechanism was proposed by Biggs et al. [7] to construct reference orbits. The method was first introduced by K. Pyragas [16], and used by Biggs et al. [7] for stabilizing halo orbits.

The feedback mechanism consists in:

\[
\begin{align*}
\dot{X}(t) &= f(X(t), t) - \nu(t) \\
\nu(t) &= -C(X(t) - X(t - \tau))
\end{align*}
\]
Constructing reference orbits (I)

- The feedback mechanism consist in:
  \[
  \dot{X}(t) = f(X(t), t) - \nu(t) \\
  \nu(t) = -C(X(t) - X(t - \tau))
  \]

- Cost function:
  \[
  J(C) = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} \nu(C, \epsilon)^2 dt
  \]

- The update rule:
  \[
  C(k+1) = C(k) - \alpha \nabla J(C)
  \]
By contrast with the methods proposed by Biggs et al. [7], the extremum seeking method is proposed to determine the K matrix gains.

The extremum seeking method showed a faster convergence.

The stability of the reference orbit.

The numerical data that describe the orbit obtained with the time-delay feedback mechanism is then fitted to a Fourier function.

The Fourier fit ensures that the reference orbit is exactly periodic.
• Biggs et al. [8] use the time-delay feedback control (TDFC) to stabilize the satellites in displaced halo orbits around the Lagrange points. Because the method is conserving the periodicity of the orbit, can be used for formation flying, but with very limited control in the relative motion of the satellites.

• For this reason we propose two other methods for the relative control, that can use the reference orbits obtained with the time-delay feedback mechanism as reference trajectories.
Orbit tracking using a PD controller

- Here we use a PD controller to track the reference orbit.

- The nonlinear system:

\[
\dot{X}(t) = f(X(t),t) + B \cdot v_{PD}(t), \\
v_{PD}(t) = -C_p(X_{xyz}(t) - X_{1,1:3}(t)) - C_d(\dot{X}_{xyz}(t) - X_{1,4:6}(t)),
\]

- To determine the control gains of the PD controller, we propose the use of the extremum seeking method, thus the control gains \(C_p\) and \(C_d\) that minimize a cost function \(J(C_p,C_d)\) must be found:

\[
J(C_p,C_d) = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} (v_{PD}(C_p,C_d,X(t),X_{1,j}(t,\tau)))^2 dt,
\]
Orbit tracking using a PD controller

- PD controller, with the determined control gains $C_p$ and $C_d$, is able to track the reference orbit and stabilize the halo orbit using a small control accelerations, thus the relative motion between the satellites in the formation is bounded, as can be seen in the figure on the left, for a period of time that will exceed the mission life. In our simulations we aimed to stabilize the motion for a period of time up to 4 years.
A sliding mode controller for relative motion control (I)

- Variational equations with a control force $u$ is considered:
  \[
  \dot{x} = w \\
  \dot{w} = m\left(\frac{x}{r^3} - 3 \frac{r \dot{r}}{r^5} \right) + u
  \]

- The aim: design a control law $u$ that will drive the relative orbit of a follower satellite with respect to a leader satellite onto a desired relative orbit.

- A new variable in the state space of the system is introduced:
  \[
  \epsilon = x - x_{ref} \\
  \dot{\epsilon} = w - w_{ref} \\
  \sigma(\epsilon, \dot{\epsilon}) = \dot{\epsilon} - c \cdot \epsilon
  \]

- In order to achieve asymptotically convergence of the state variables in the presence of bounded disturbances, the variable $\sigma$ must be driven to zero by means of control $u$.

- The $\sigma$ dynamics are derived:
  \[
  \dot{\sigma} = c \dot{\epsilon} + f(\epsilon, \dot{\epsilon}) + u
  \]

- The Lyapunov candidate function is chosen:
  \[
  V = \frac{1}{2} \sigma^2
  \]
To provide asymptotic stability the following conditions must be satisfied:

(i) $\dot{V} < 0 \text{ for } \sigma \neq 0$

(ii) $\lim_{|\sigma|} V = \infty$

Resulted control function $u$ is:

$$u = -c \dot{\sigma} - \rho \text{sign } \sigma$$
In the numerical simulations the following considerations were made:

- \( \mu = 1 \)
- A circular relative orbit was tracked. Naturally there are no circular relative orbits, thus the follower satellite is constantly forced to have a relative circular orbit with respect to the leader.
- The control thrust should not exceed 0.009\( \text{m/s}^2 \) (300\( \text{mN} \) on a 500\( \text{kg} \) S/C).
A sliding mode controller for relative motion control (III)

- The relative orbit error is in the order of cm.
- The control acceleration does not exceed thrust levels that SEP deliver.
- Compared to the relative motion behavior, it can be seen that the SMC stabilizes and maintains the desired relative motion of the satellites.
Summary

- Families of same period artificial halo orbits around the Lagrange points L1 and L2 have been found.
- The small and medium orbits in the halo families are unstable, thus active control is required to accurately maintain the formation.
- Reference orbits where designed using the time-delay feedback mechanism and track them with a PD controller.
- The gains of the time-delay feedback mechanism and of the PD controller are determined using the extremum seeking method.
- An SMC is proposed for an accurate relative motion control.
- It is shown that stable formation flying around the Lagrange points can be achieved using solar electric propulsion.
Future Work

- Use the time-delay feedback control to control the relative motion of satellites.
- Use DDE tools to study the stability of relative motion control with time-delay feedback.
- Multiple delay controller for smoothing noise.
References

Thank you!