The Full Problem of Two and Three Bodies: Application to Asteroids and Binaries

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Motivation:

- Increased interest in asteroids
  - remnant debris of formation of S.S
  - some are near the Earth $\Rightarrow$ small $\Delta v$
  - threat of impact with the Earth
- Asteroids are not spherical $\Rightarrow$ allow for richer dynamics

- Presence of binaries $\Rightarrow$ coupled dynamics
- Non-linear dynamics can be exploited:
  - real missions
  - understand the formation of S.S.
Aims of the research:

Study and understand the following problems:

- Dynamical environment of rotating non-spherical bodies
- Dynamics of binary and multiple systems
- Dynamical environment of a binary system
Aims of the research:

Asteroids have very irregular non-spherical shapes

Study the models that take shape into account
Objectives:

- Development gravitational model & comparison with others
- Study transport of material
- Landing on asteroids using non-linear dynamics
- Approximation equilibrium points of binary asteroids
- Comparison RF3BP vs RTBP
- Formation and evolution of asteroids
Assumptions: Body of which \( M \) and spherical harmonics coefficients are known:

Developed a gravitational potential such that it is ...

- smooth everywhere and composed of two parts
  - external: spherical harmonics
  - internal: spherical Bessel functions
- easy to compute (computed on-board a spacecraft)
- able to reproduce particular dynamics such as location eq. points

\[
M \quad \omega \\
I_{xx} \quad I_{yy} \quad I_{zz}
\]
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External/internal gravitational potential
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$$\Phi_e = \frac{G}{R} \sum_{n=0}^{\infty} \left( \frac{R}{r} \right)^{n+1} \sum_{m=0}^{n} P_n^m(\cos(\theta)) \left( a_{nm} \cos(m\varphi) + b_{nm} \sin(m\varphi) \right)$$

$$\Phi_i = \frac{G}{R} \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{n} j_n \left( \frac{\alpha_{ln} r}{R} \right) P_n^m(\cos(\theta)) \left( A_{lnm} \cos(m\varphi) + B_{lnm} \sin(m\varphi) \right)$$
Gravitational potential
spherical Bessel functions and spherical harmonics

Asteroid Eros modelled with:

- polyhedron potential
- spherical harmonics 4th order
- expansion external + internal 4th order
Gravitational potential using spheres

- Spherical bodies with constant $\rho$ can be reduced to point masses
  \[ U_i = \frac{Gm_i}{r_i} \]
- The body is modelled using cotangent spheres $U = \sum_i U_i$
- The shape and location of spheres given by skeletonization
Gravitational potential using spheres

Asteroid Castalia: external/internal and 2 spheres with different $\rho$

(a) $\delta=0.8$
(b) $\delta=1.0$
(c) $\delta=1.2$
(d) $\delta=1.5$

(e) $\delta=0.8$
(f) $\delta=1.0$
(g) $\delta=1.2$
(h) $\delta=1.5$
Dynamics: Dynamical environment of a non-spherical asteroid

1. Using potential developed up to second order
   - study equilibrium points
   - study of periodic orbits and their manifolds
   - study of dust behaviour

2. Using spheres potential
   - study of equilibrium points and bifurcations
Dynamics: Dynamical environment of a non-spherical asteroid

Equilibrium points: Parameters: \( \alpha_x = \frac{l_{xx}}{l_{zz}} \quad \alpha_y = \frac{l_{yy}}{l_{zz}} \quad \omega \)
Periodic orbits:

- There exists a family of unstable periodic orbits emanating from the long axis equilibrium point.
- There exist two families of stable periodic orbits around short axis equilibrium point when it is stable.
- There can exist periodic orbits around the short axis equilibrium point when it is unstable.
Dynamics: Dynamical environment of a non-spherical asteroid

Periodic orbits:

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Dynamics: Dynamical environment of a non-spherical asteroid

**Manifolds:**

- The stable manifold is a set of trajectories that asymptotically approaches the periodic orbit.
- The unstable manifold is a set of trajectories that asymptotically departs from the periodic orbit.
Dynamics: Dynamical environment of a non-spherical asteroid

- We have followed the manifolds of periodic orbits until:
  - escape
  - impact on body
  - a maximum time is reached
- For the ones that impact the body: angles $\phi$ and $\alpha$
Dynamics:
Dynamical environment of a non-spherical asteroid

Classification of manifold trajectories

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Full Problem of 2 & 3 Bodies
Dynamics:
Dynamical environment of a non-spherical asteroid

Dust behaviour:
- Integration of particles leaving from the surface $\phi \in [0, \pi]$ and $\alpha \in [0, \pi]$ for a fixed energy
- Classification of behaviour

![Graphical representation of dust behaviour](image)
Dynamics: Landing on asteroids

- We want to use the manifolds to approach, observe and land
- Some trajectories orbit for a long time around the asteroid before escaping or impacting $\implies$ good for observing the body
- Trajectories that impact on the body are very tangential!
Dynamics:
Landing on asteroids

- Implement a control that forces spacecraft to follow confocal hyperbolas
- The hyperbolas are orthogonal to the confocal ellipses
Dynamics:
Landing on asteroids

**Table:** Increments of velocity and time of flight for a landing manoeuvre on asteroid Nereus.

<table>
<thead>
<tr>
<th>Leg</th>
<th>$\Delta v$</th>
<th>TOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>insertion st. man.</td>
<td>$0.01772 \text{ ms}^{-1}$</td>
<td>14.1 days</td>
</tr>
<tr>
<td>jump unst. man.</td>
<td>$\approx 0.0 \text{ ms}^{-1}$</td>
<td>-</td>
</tr>
<tr>
<td>vertical landing</td>
<td>$0.1132 \text{ ms}^{-1}$</td>
<td>8.94 days</td>
</tr>
</tbody>
</table>

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FULL PROBLEM OF 2 & 3 BODIES
Dynamics: The full problem of two bodies

Definition of the problem

- Two asteroids rotating around their common barycentre
- Assume one is spherical
- The elongated asteroid modelled with expansion up to 2nd order
- Two integrals of motion: $E$ and $K$
Dynamics:
The full problem of two bodies

Equilibrium dynamics

E = -0.05
$\alpha_x = 0.4$ $\alpha_y = 0.6$ $\nu = 0.5$

- long axis eq is energetically stable
- binary asteroids found in long axis equilibrium
Dynamics:
The full problem of two bodies

Non-equilibrium dynamics

- periodic orbits
- pendulum approximation
Dynamics: The Restricted Full Three Body Problem

1. Primaries in equilibrium
   - Equilibrium points & their stability
   - Dynamics around $L_3$: horseshoe orbits

2. Primaries near equilibrium
   - Study frequencies of oscillation
   - Osculating Lagrange points
1. Primaries in equilibrium
   - Equilibrium points & their stability
   - Dynamics around $L_3$: horseshoe orbits
Equilibrium points & their stability
Analytical proofs

Long axis case
The collinear points have always a saddle-centre behaviour

Short axis case
The collinear points $L_1$ and $L_2$ have always a saddle-centre behaviour. $L_3$ can have saddle-centre, unstable focus or centre-centre behaviour.
Dynamics: The Restricted Full Three Body Problem

Dynamics around $L_3$: horseshoe orbits

- For a given mass and energy we study how the shape of the body affects the number of horseshoe orbits
- Function that every crossing at 0 gives a horseshoe orbit
Dynamics: Non-linear dynamics and asteroid evolution

Using the spheres potential for simplicity...

- Consider families of p.o
- Integrate manifolds until a circle of radius \( r \)
- Uniform set of initial conditions on the circle
Consider a grid overlaid on top of the model.

At every fixed time step, we increment the counter on the cell.

It gives us a measure of density.

Accumulation near equilibrium points.
Growth of secondary bodies
Dynamics: Non-linear dynamics and asteroid evolution

- Secondary bodies grow from material from the hot disk

- Two things can happen
  - the secondaries touch the initial body $\Rightarrow$ change of inertia and rotation $\Rightarrow$ change position of equilibrium points
  - the secondaries separate

- In both cases the procedure can be repeated
- Up to when?
Dynamics:
Non-linear dynamics and asteroid evolution

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Full Problem of 2 & 3 Bodies
Conclusions

- It is possible to model the gravitational potential
  - in an easy and computationally efficient way
  - such that it satisfies dynamical constraints
- The stable manifolds of periodic orbits classify the short term behaviour of ejecta
- It is possible to design landing trajectories that make use of the invariant manifolds
- Analytical models can be derived that approximate the coupling between orbital and rotational motion for binary asteroids
Conclusions

- The RF3BP can be very different to the RTBP when primaries are in short axis equilibrium configuration.
- The number of horseshoe orbits decreases when the body becomes more elongated.
- The 3 time-scales present in the RF3BP are of comparable size, which makes the dynamics difficult to predict.
- The non-linear dynamics about unstable equilibrium points can focus material on particular regions.
- During the S.S formation period this focusing behaviour could have had a role in the evolution of asteroids.
Questions?
Questions?

Thank you!