Consider a dynamical system 
\[ \dot{x} = f(x, t) \quad x \in \mathbb{R}^n \]
and define its flow map \( \varphi(t; t_0, x_0) \) as the map that give the position of a particle which at time \( t_0 \) is at \( x_0 \).

The objective of the Jet Transport is to compute \( \varphi(t; t_0, U) \) where \( U \) is a full neighborhood of initial conditions parametrized by a polynomial \( P_{k,n}(\xi) = x_0 + \xi \).

The main ingredients of a Jet Transport procedure are:
- polynomial algebra including addition, products, elementary functions... of polynomials in \( n \) variables
- integration method any integration method can be used (RK, Taylor, Symplectic,...).

Example
Consider a linear center: \( (\dot{x}, \dot{y}) = f(x, y) = (y, -x) \)
The neighbourhood \( U \) of the initial condition \( (x(0), y(0)) = (x_0 + \xi_1, y_0 + \xi_2) \)
Apply a step of Euler’s method
\[
(x_{n+1}, y_{n+1}) = (x_n, y_n) + h \cdot f(x_n, y_n)
\]
Since we are implementing the Jet Transport, the new state depends on the variables \( \xi_1 \), and is equal to
\[
(x_1, y_1) = \left( x_0 + \xi_1 + h \cdot (y_0 + \xi_2), y_0 + \xi_2 - h \cdot (x_0 + \xi_1) \right) = \left( x_0 + h \cdot y_0, y_0 - h \cdot x_0 \right) + \left( 1, -1 \right) \frac{1}{h} \xi_2
\]

Satellite Collisions: a colliding scenario with some close approaches

Let us study two satellites which have close encounters at times \( t_0, t_1 \) and \( t_2 \) and collision at a time \( t = t_2, t_1 < t_2 < t_0 \).

Algorithm
1. For each satellite, take as \( U \) a sphere around the initial conditions
2. compute the Jet transport of each \( U \) at the epochs \( t_i \)
3. Choose random initial conditions in the two initial spheres (using uniform or Gaussian distribution)
4. For each pair of initial conditions, compute the closest approach for times close to the epochs \( t_i \)
5. Determine the distance between both image points
6. Repeat steps 3-5 for the full set of initial data (10^6 pairs).

<table>
<thead>
<tr>
<th>Approach</th>
<th>mean absolute deviation</th>
<th>skewness</th>
<th>kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>49041.81</td>
<td>92.760</td>
<td>0.010</td>
</tr>
<tr>
<td>Second</td>
<td>47710.97</td>
<td>141.445</td>
<td>0.034</td>
</tr>
<tr>
<td>Third</td>
<td>545.739</td>
<td>359.16</td>
<td>0.634</td>
</tr>
<tr>
<td>Fourth</td>
<td>44475.29</td>
<td>376.657</td>
<td>0.045</td>
</tr>
</tbody>
</table>

Red: Histograms of frequencies of the minimum distance for four different close approaches between the spacecraft. Blue: normal distribution with same mean and standard deviation.

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LCS Substitutes

In the early 2000 G. Haller [1] defined the Lagrangian Coherent Structures (LCS) as structures that give some information on the structure of the system. They are defined as curves following ridges of the Finite Time Lyapunov Exponents (FTLE) scalar fields \( \sigma_k(\bar{x}) = \frac{1}{h} \ln \frac{\|d\bar{x}/dx\|}{\|d\bar{x}/dx\|} \).

If two initial orbits are close after some time, the FTLE at that time will be small, otherwise, if they are far away the FTLE will be large. Therefore, curves with high FTLE will give regions where there are different behaviours.

FTLE fields in the RTB: forward (left), backward (right) integrations. Invariant manifolds: stable (green), unstable (red).

Bibliography