Endomorphism algebras of geometrically split abelian surfaces over $\mathbb{Q}$

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A conjecture and a particular case

- A abelian variety over a number field k
  - $\mathrm{End}^0_{\overline{\mathbb{Q}}}(A) = \mathrm{End}(A_{\overline{\mathbb{Q}}}) \otimes \mathbb{Q}$ the algebra of $\overline{\mathbb{Q}}$-endomorphisms
- For $g, d \geq 1$ define

$$\mathcal{A}_{g,d} = \{\mathrm{End}^0_{\overline{\mathbb{Q}}}(A) \colon A/k \text{ of dimension } g \text{ and } [k:\mathbb{Q}] = d\}$$

Conjecture (Coleman)

The set $\mathcal{A}_{g,d}$ is finite.

- We are interested in $\mathcal{A}_{2,1}$: possible endomorfism algebras of abelian surfaces $A/\mathbb{Q}$
  - If $A_{\overline{\mathbb{Q}}}$ is simple $\rightsquigarrow$ completely open
  - We are interested in the cases where $A_{\overline{\mathbb{Q}}}$ is not simple

A theorem

The set $\mathcal{A}_{2,1}^{\text{split}}$ of $\overline{\mathbb{Q}}$-endomorphism algebras of geometrically split abelian surfaces over $\mathbb{Q}$ is made of:

1. $\mathbb{Q} \times \mathbb{Q}$, $\mathrm{M}_2(\mathbb{Q})$;
2. $\mathbb{Q} \times K$, $K \times K'$, where K, K' are quadratic imaginary fields of class number 1;
3. $\mathrm{M}_2(K)$ where K is a quadratic imaginary field with class group 1, $\mathbb{Z}/2\mathbb{Z}$, or $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ and distinct from

$$\mathbb{Q}(\sqrt{-195}), \mathbb{Q}(\sqrt{-312}), \mathbb{Q}(\sqrt{-340}), \mathbb{Q}(\sqrt{-555}), \mathbb{Q}(\sqrt{-715}), \mathbb{Q}(\sqrt{-760})$$

In particular, the set $\overline{\mathcal{A}}_{2,1}$ has cardinality 92.