Chapter 4

Fatigue Model Description

4.1 Introduction

The human body is continuously under the influence of external or internal forces. The application of forces may result in beneficial effects (i.e. bone remodeling) or prejudicial ones (i.e. fractures). Fatigue can be considered as a decrease in physical performance due to external and internal forces. We aim at modeling fatigue apparition due to the stress produced by body segments' mass and external loads.

4.2 Fatigue at Joint Level versus Fatigue at Muscle Level

Fatigue assessment can be considered from two points of view: fatigue assessment at joint level or fatigue assessment at muscle level.

Fatigue at joint level evaluates the fatigue produced by a group of muscles acting on a joint. External forces acting on a body segment under gravity produce load moments at body joints. These load moments can be compared to muscle strength moments and thus provide a means to evaluate how stressed joints are. Strength equations provide us with the muscle-produced moment strength at each joint. Because muscles often span two (or more) joints, the angle at adjacent joints is considered in some strength values prediction.

In a joint as complex as the shoulder, its motion in the sagital plane is produced by a group of muscles that also contribute to other joint movements such as the elbow joint.

Assessing fatigue at muscle level is a tedious task as the human body contains many individual muscles. The movement of the sole shoulder joint is produced by fifteen muscles and as mentioned before, a single muscle can participate in some movements as the prime mover and in other as assistant muscle. As mentioned in the previous chapter, Komura used an individual muscle model of the leg [Kom97] whereas we use a fatigue model at muscle groups level because the joint space is much more intuitive for end-user control compared to the muscle state space.

4.3 Modeling Antagonistic Muscle Groups: the Half-joint Concept

We propose to calculate fatigue at joint level, more precisely at muscle group level [Rod03b]. To do that, we need the concept of half-joint pair. To better understand the half-joint concept, we need to know about the parameters used to calculate fatigue at joint level. A joint fatigue value depends on two factors: current joint torque and maximum voluntary contraction (strength).

Some biomechanics studies have established the relationship between a joint value and the corresponding strength for a muscle group [Kul84]. Strength in several groups of men and women has been collected in experiments and regression analysis has been done to obtain strength equations [Cla66]. Chaffin presented mean strength prediction equations summarizing the main joints of a male population [Cha88]. These equations are shown in the next section.

It is important to realize that for each degree of mobility, two groups of muscles acting in opposite directions have to be considered. The reason is that muscles can pull but cannot push, thus leading to assemble them in two groups of antagonist pulling muscles for each degree of mobility [Kat82]. Therefore, to move rigid elements like bones, muscles must work in pairs. Our proposal is to split each single *dof* joint (*dof* is short for degree of freedom) into two coordinated half-joints, thus the name of half-joint pair. Each half-joint reflects the activity of the group of muscles associated to one degree of freedom, in one direction*:* the pulling direction of the corresponding muscle group. In Anatomy, the two muscle groups constituting the half-joint pair are said to be antagonist. Based on this organization, we can calculate and visualize independent fatigue variables for each antagonist muscle group. The half-joint pair organization smoothly integrates within our Inverse Kinematics framework.

A single *dof* joint can be represented by the scalar value of its parameter, noted θ_{cur}, within the allowed range of motion [θ_{min}, θ_{max}]. A half-joint pair is built by splitting the joint's range into two complementary intervals, around the current joint value θ_{cur}.

For example, the elbow flexion and extension movements are produced by antagonist groups of muscles. That is the reason why there are two different muscular strengths for the elbow in the sagital plane. In Figure 4-1 elbow joint is broken down into two joints: θ_{E1} and θ_{E2} whose range of movement is defined as follow:

half-joint θ_{E1}: range of movement defined by [$\theta_{E\text{-}min}$, $\theta_{E\text{-}cur}$]

half-joint θ_{E2}: range of movement defined by [$\theta_{E\text{-}cur}$, $\theta_{E\text{-}max}$], where $\theta_{E\text{-}cur}$ evolves over time within its inferior and superior limits $\theta_{E\text{-}min}$ and $\theta_{E\text{-}max}$, half-joint θ_{E1} represents the flexion and half-joint θ_{E2} represents the extension.

As the half-joint pair is structured in a parent-child nodes hierarchy, their range of movement becomes [$\theta_{E\text{-}cur}$, $\theta_{E\text{-}max}$] for the parent half-joint and [0 , $\theta_{E\text{-}cur}$ -$\theta_{E\text{-}min}$] for the child half-joint. Note that half-joints have dynamic limits since $\theta_{E\text{-}cur}$ changes when the elbow posture changes. Note also that both joints are defined with opposite axes of rotation.

4.4 Fatigue Model Parameters

Fatigue model parameters are joint strength and the current joint torque, which are used to calculate a value of normalized torque [Rod02] (See Figure 4-2). The normalized torque is used to compute the maximum holding time that the posture can be sustained (in an evolving static context). Finally, the current value of the maximum holding time and the time increment are considered to update joint fatigue level. This process operates while trying to achieve user-defined goals with an Inverse Kinematics solver.

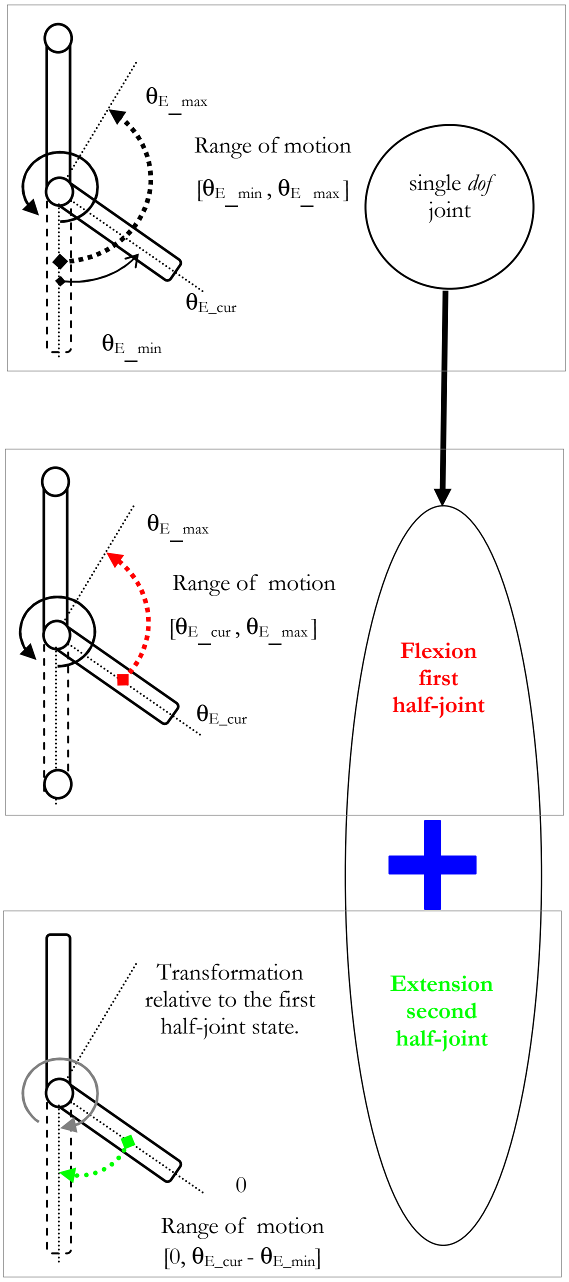


Figure 4-1. A single *dof* joint (the elbow) is split in two half-joints (a half-joint pair)

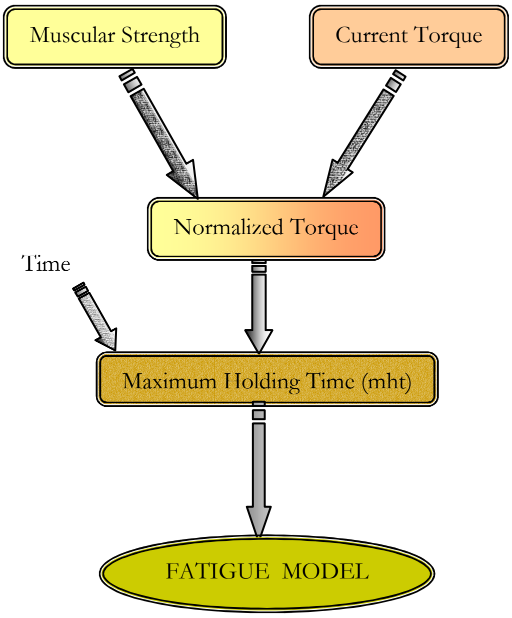


Figure 4-2. Fatigue model parameters

4.4.1 Muscular Strength

Muscular strength is the maximum capacity that a group of muscle has to produce force. The simplest way to relate loading moment to maximum capacity (muscular strength) is to normalize it by the latter. Most studies express their findings in these normalized quantities.

The main function of a muscle is to produce a force along the muscle line of action. Depending on the current posture which influences the muscle lever arm, this force causes a moment with respect to a joint axis (See Figure 4-3). This explains that, for a group of muscles acting in a given direction (e.g. flexion or extension), the maximal exertable torque is not constant but depends on the current posture.

As recalled earlier, the strength depends on the posture. Some biomechanics studies have established the relationship between a joint value and the corresponding strength for a muscle group. At that stage it is important to note that for each degree of mobility, two groups of muscles acting in opposite directions have to be considered. There are three types of strength curves as illustrated on Figure 4-4: ascending, descending and ascending-descending.

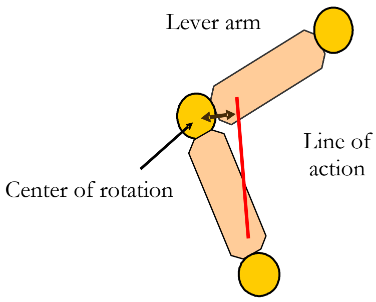


Figure 4-3. The lever arm of the muscle depends on its line of action

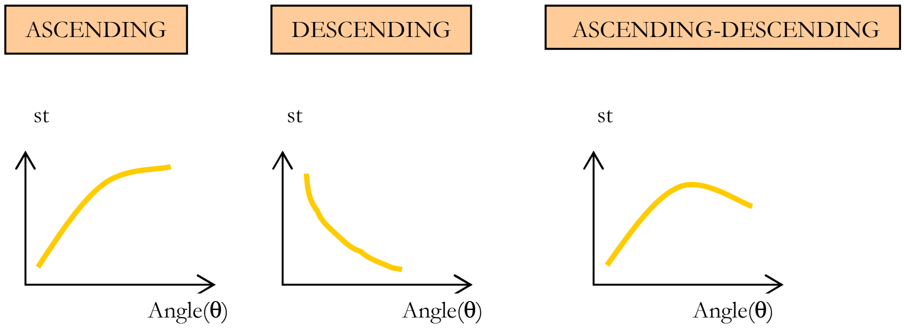


Figure 4-4. Types of strength curves (from [Kul84])

Table 4-1 lists mean strength prediction equations and type of strength curve for the main joints of the human body [Cha88]. We have based our joint strength model on Chaffin results. Chaffin gathered data from other studies [Bur72] [Sha72] [Sto82].

STRENGTH st = g(θ)	JOINTS	EQUATION (units: Nm)	TYPE OF CURVE
Elbow flexion	Elbow and shoulder	$(336.29+1.544\theta_E-0.008\theta_E^2-0.5\theta_S)\ 0.1913$	Asc-desc
Elbow extension	Elbow and shoulder	$-(246.153-0.575\theta_E-0.425\theta_S)\ 0.2126$	Descend.
Shoulder flexion	Shoulder and elbow	$(227.338+0.525\theta_E-0.296\theta_S)\ 0.2845$	Ascend.
Shoulder extension	Shoulder	$-(204.562-0.099\theta_S)\ 0.4957$	Asc-desc
Seated torso flexion	L5/S1	$-(141.179 + 3.694\theta_T)\ 0.2796$	Asc-desc
Seated torso extension	L5/S1	$(3365.123 - 23.947\theta_T)\ 0.3381$	Asc-desc
Standing torso flexion	L5/S1	$-(17.17\theta_T - 0.079\theta_T^2)\ 0.2146$	Asc-desc

Standing torso extension	L5/S1	$(3894 - 13.9\theta_T)\ 0.1559$	Asc-desc
Hip flexion	Hip	$(-820.21+34.29\theta_H-0.11426\theta_H^2)\ 0.1304$	Ascend.
Hip extension	Hip	$-(3338.1-15.711\theta_H+0.04626\theta_H^2)\ 0.0977$	Descend.
Knee flexion	Knee	$-(-94.437+6.3672\theta_K)\ 0.1429$	Ascend.
Knee extension	Knee	$(1091.9-0.0996\theta_K+0.17308\theta_K^2)\ 0.0898$	Asc-desc
Ankle extension	Ankle	$-(3356.8-18.4\theta_A)\ 0.0816$	Asc-desc

Table 4-1. Strength as function of joint angles: Elbow θ_E, Shoulder θ_S, Torso θ_T, Hip θ_H, Knee θ_K, Ankle θ_A

Angles used in strength equations are those showed in Figure 4-7. Note that a joint angle is measured using the angle between adjacent bones. In particular, torso angle is measured from the end of the spine (L5/S1, lumbo-sacral joint), indicated by the discontinuous line. As can be observed, hip angle is measured from a forward position.

Figure 4-5 and Figure 4-6 show the graphical representation of elbow strength equations presented in Table 4-1.

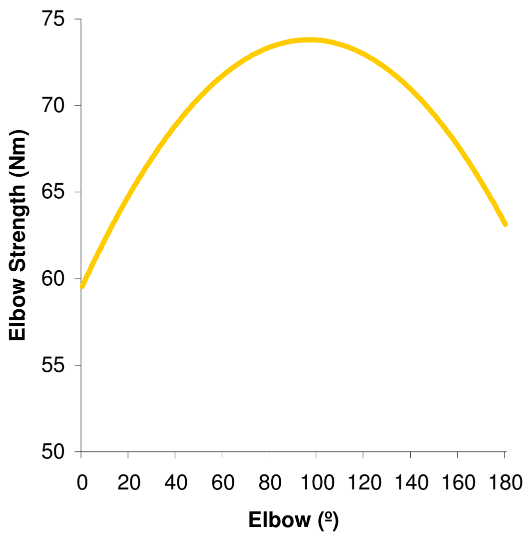


Figure 4-5. Elbow flexion strength

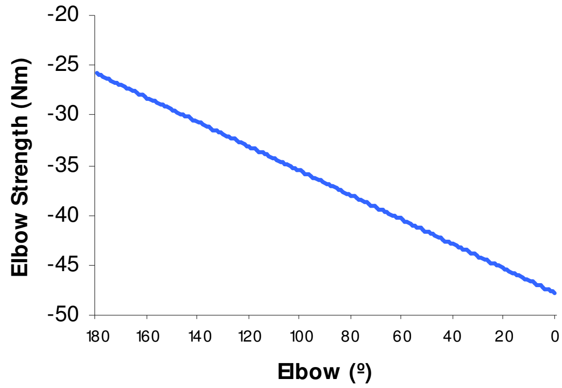


Figure 4-6. Elbow extension strength

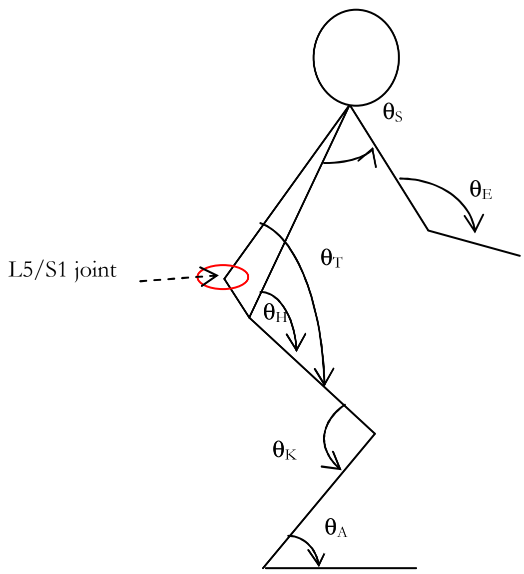


Figure 4-7. Angle conventions (from [Cha88])

4.4.2 Torque Computation

In the torque computation, we consider active and passive torques. Active torque is produced by external loads and by the own body mass. Ligaments and connective tissues produce passive torque, which mainly appears at the extremes of the joint range of motion.

4.4.2.1 Active Torque

A loading moment is the moment, with respect to a specific motion axis, caused by the body segments mass and by external forces. This moment is counteracted by a moment of equal magnitude but opposite direction exerted by the active muscles. Loading moments can be used to describe the load over time, to compare different ways of performing a task or to study the effects of various means of load reduction.

We work under the static equilibrium assumption, so the joint torques due to external loads can be computed from the Jacobian transpose using the principle of virtual work [Cra86]. The transpose of the Jacobian maps cartesian forces actuating at an effector into equivalent joint torques:

$$\tau_{external} = \sum_i J_i^T F_i + J_G^T P$$

where F_i represents an external Cartesian force, P represents the body weight acting at the center of mass, J_i^T the Jacobian transpose for the end effector *i* and J_G^T is the Jacobian used to the control of the center of mass [Bou96].

The current torque applied to a joint is expressed as the sum of internal and external torques:

$$\tau = \tau_{internal} + \tau_{external}$$

Internal torque includes active and passive ones. As we work under the static case, the total torque must be equal to zero:

$$(\tau_{active} + \tau_{passive}) + \tau_{external} = 0$$

Then, active torque is expressed as follows:

$$\tau_{active} = -\tau_{external} - \tau_{passive}$$

4.4.2.2 Passive Torque

The passive component appears when a joint is forced in a certain direction toward its limits. Several researches have published data which characterize the passive torques developed at each joint [Yoo82][Man86][Dig95]. Figure 4-8 displays the general characteristic of the passive behavior.

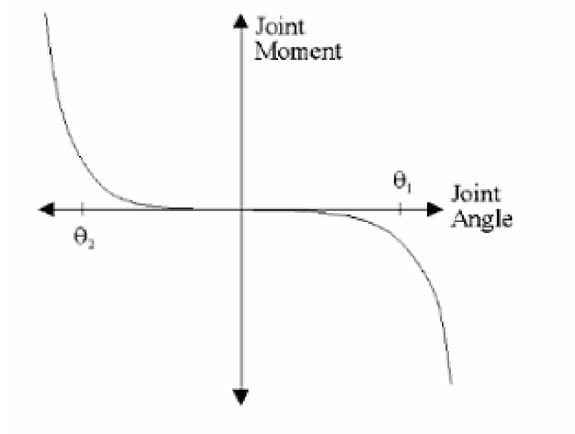


Figure 4-8. Passive joint moment vs. joint angle (from [Dig95])

There is no general mathematical approach to describe the passive joint moment. However, experimental studies have tried to measure it. Some of them considered the influence of adjacent joint angles. The goal of Riener's study was to measure the passive elastic joint moment over a wide range of positions of hip, knee and ankle joint, while also taking into account the influence of the adjacent joint angles [Rie99].

Riener proposed a simple mathematical model, which can be used to estimate the passive elastic joint moments as a function of lower limb angles. The model is defined as follows:

$$\tau_{passive\_A} = \exp(2.1016 - 0.0843\theta_A - 0.0176\theta_K) - \exp(-7.9763 + 0.1949\theta_A + 0.0008\theta_K) - 1.792$$

$$\tau_{passive\_K} = \exp(1.800 - 0.0460\theta_A - 0.0352\theta_K + 0.0217\theta_H) - \exp(-3.971 - 0.0004\theta_A + 0.0495\theta_K - 0.0128\theta_H) - 4.820 + \exp(2.220 - 0.150\theta_k)$$

$$\tau_{passive\_H} = \exp(1.4655 - 0.0034\theta_K - 0.0750\theta_H) - \exp(1.3403 - 0.0226\theta_K + 0.0305\theta_H) + 8.072$$

where $\tau_{passive\_A}$, $\tau_{passive\_K}$ and $\tau_{passive\_H}$ represent passive torque at ankle, knee and hip joints, and $\theta_A, \theta_K, \theta_H$ are ankle, knee and hip angles in degrees.

While this model accounts for passive muscle properties described as total passive joint moments, Zajac used a musculo-tendon model in which passive and active forces were generated by single muscles [Zaj89]. Zajac's model has many parameters that cannot be identified non-invasively due to the muscle-joint redundancy of the musculoskeletal system. We adopt the model found in Figure 4-8.

Figure 4-9 shows a comparative of measured and predicted by the model joint angle-passive moment curve of the knee.

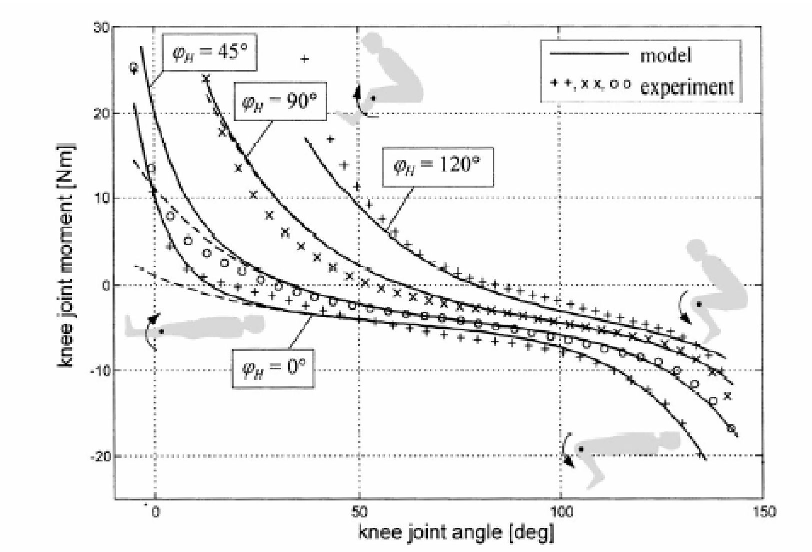


Figure 4-9. Measured and predicted passive moment at knee joint (from [Rie99])

4.4.3 The Maximum Holding Time

Manenica stated that there was a relationship between the force a muscle exerts and the maximum holding time that the posture can be maintained [Man86]. The maximum holding time *mht* is a function of the normalized torque T_N:

$$mht = f(T_N)$$

Normalized torque is calculated as the quotient of joint active torque and joint strength:

$$T_{N_i} = \frac{\tau_{active_i}}{st_i}$$

where T_{N_i}, τ_{active_i} and st_i represent respectively normalized torque, current active torque and strength of joint *i*.

An experimental study of several groups of male and female highlighted that there were not so many differences between force-time relationships in different muscle groups. Therefore, a general force-time relationship expressed as a regression line, valid for several muscle groups, was obtained:

$$mht_i = \exp(2.70 - 0.0448 \ T_{N\,i}) \qquad (1)$$

where mht_i represents the maximum holding time that joint i is able to maintain the posture and $T_{N\,i}$ represents the normalized torque. Time is expressed in minutes.

Table 4-2 shows data obtained by Manenica, the relationship between the posture holding time and the muscular force applied, expressed as a percentage of the maximal muscle force (strength).

	Force (%) 20	40	60	80	100
Muscles involved	(posture max. holding time in minutes)				
Hand grip	6.17	2.50	1.02	0.41	0.17
Biceps brachii	6.17	2.50	1.02	0.41	0.17
Body pull	4.85	2.10	0.90	0.39	0.17
Body torque	5.26	2.22	0.94	0.40	0.17
Back muscles	8.50	3.19	1.20	0.45	0.17

Table 4-2. Force-time relationship for different muscle groups (from [Man86])

4.5 A Variational Expression of Fatigue

By definition the fatigue level is expressed as the holding time *ht* normalized by maximum holding time *mht*:

$$fatigue\_level = \frac{ht}{mht} \qquad (2)$$

We consider a more general case than studies that work with an a priori knowledge of motion evolution through time because we allow the posture to change slowly over time. In that context, we still want to estimate how the fatigue evolves over the elapsed time. For that purpose we propose a variational expression of fatigue where the fatigue variation is a function of the time step duration Δt and of the current maximum holding time *mht*:

$$\frac{ht_t}{mht} = \frac{ht_{t-1}}{mht} + \frac{\Delta t}{mht} \qquad (3)$$

Formula (3) defines fatigue level at time t as the fatigue level at time t-1 plus a term that expresses an additional fatigue increment produced during Δt. In such a way, we extend the scope of formula (2) due to the possible variation of *mht* over time depending on the context (calculated with (1)).

$$fatigue\_level_t = fatigue\_level_{t-1} + \frac{\Delta t}{mht}$$

Several studies have concluded that fatigue appears when the normalized torque is above a threshold of 15% of the muscular strength [Roh60]. For this reason, we propose to scale the fatigue increment with a factor, see Figure 4-10, representing the gradual manifestation of fatigue. This is the fatigue factor F_F as shown in the following equation:

$$fatigue\_level_t = fatigue\_level_{t-1} + \frac{\Delta t}{mht} F_F \qquad (4)$$

The following graphic represents F_F:

$$0. \le T_N \le 0.15: \qquad F_F = 6.6 T_N$$
$$0.15 \le T_N \le 1.: \qquad F_F = 1$$

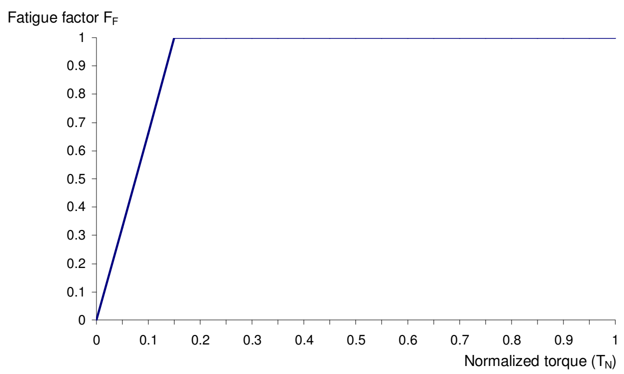


Figure 4-10. Fatigue factor as a function of the normalized torque

4.5.1 The Recovery Term

Equation (4) is now extended with a negative term, called the recovery decrement, approximating the effect of a static recovery produced by a period of rest. Milner carried out an investigation to observe the relationship between holding time, rest and recovery [Mil86].

We define the minimum duration of recovery *mdr* as the maximum holding time multiplied by the current fatigue level.

Therefore, we exploit formula (1) to evaluate *mdr* using a floating average of the torque to reflect its recent history. The window size used to compute the floating average is a numerical value that depends on the current instantaneous torque (See Figure 4-11). If the instantaneous torque is very low, we use a wide window to compute the average torque. The maximal window size corresponds to the longest value of maximum holding time. For higher instantaneous torques, the window size decreases to be more sensitive to the recent past. In this case, the window size is given by the maximum holding time calculated using the current instantaneous torque.

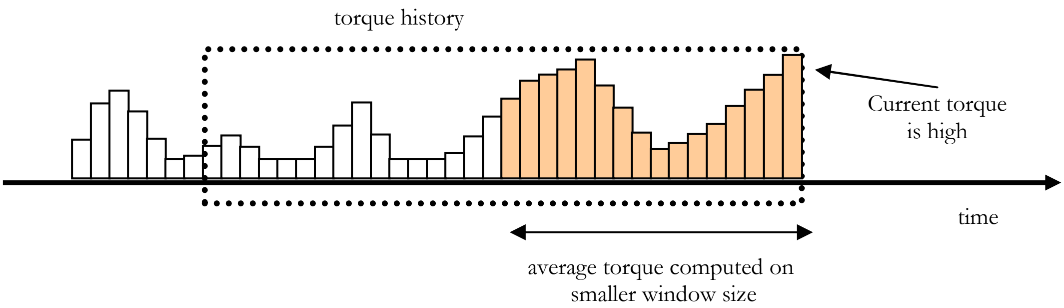
This approach avoids discontinuities when after a rest period with null torque, a sudden instantaneous high torque just shortens the window size for average torque computation, in this way average torque raises continuously.

Finally, for even higher values the recovery term is filtered out by a recovery factor R_F which is a function of the normalized torque T_N (See Figure 4-12). The idea is that recovery appears only for small values of the normalized torque, so R_F is defined as follows:

$$0.15 >= T_N >= 0: \quad R_F = 1 - 6.6T_N$$
$$1.0 >= T_N > 0.15: \quad R_F = 0$$

With the recovery term, the final formulation of fatigue level is the following:

$$\boxed{fatigue\_level_t = fatigue\_level_{t-1} + \frac{\Delta t}{mht} F_F - \frac{\Delta t}{mdr} R_F}$$



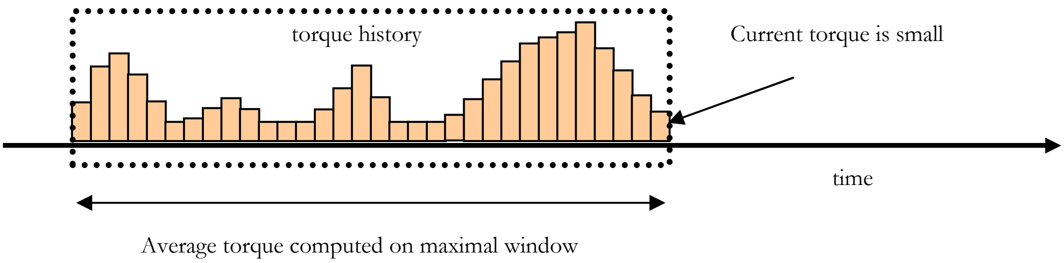


Figure 4-11. Average torque computation

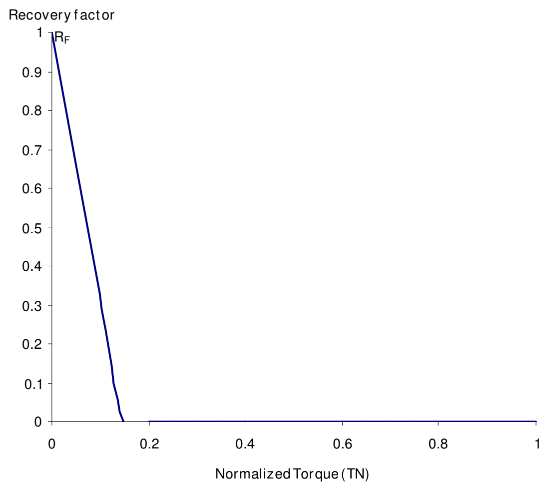


Figure 4-12. Recovery factor as a function of the normalized torque

4.5.2 The Entire Process of Fatigue Calculation

Figure 4-13 outlines the process of fatigue level calculation. Each of the six following paragraphs summarizes each step (shape in Figure 4-13) in the fatigue model formulation:

- Input parameters are muscular strength and active current joint torque. Joint strength value st is obtained from strength curves and current joint torque is computed from the Jacobian transpose using the principle of virtual work.

- The minimal duration of recovery *mdr* is calculated using a floating average of the torque to reflect its recent history.

- For each joint *i* the normalized torque $T_{N\,i}$ is calculated as the quotient of joint torque τ_i and joint strength st_i. A study on maximum holding time that a posture can be sustained stated it as a function of normalized torque.

- Then fatigue level is expressed as the holding time *ht* normalized by the maximum holding time *mht*.

- As we want to estimate how fatigue evolves over the elapsed time, a variational expression of fatigue is defined where the fatigue variation is a function of the time increment duration Δt and of the current maximum holding time *mht*. Fatigue level at time k is defined as fatigue level at time k-1 plus a term that expresses an additional fatigue increment produced during Δt.

- The final formulation of fatigue is given in the last shape. Rohmert estimated that fatigue apparition is produced when the normalized torque is above a threshold of 15% of the muscular strength. We propose to scale the fatigue increment with a fatigue factor F_f representing the gradual manifestation of fatigue. A negative term approximates the effect of a static recovery produced by a period of rest.

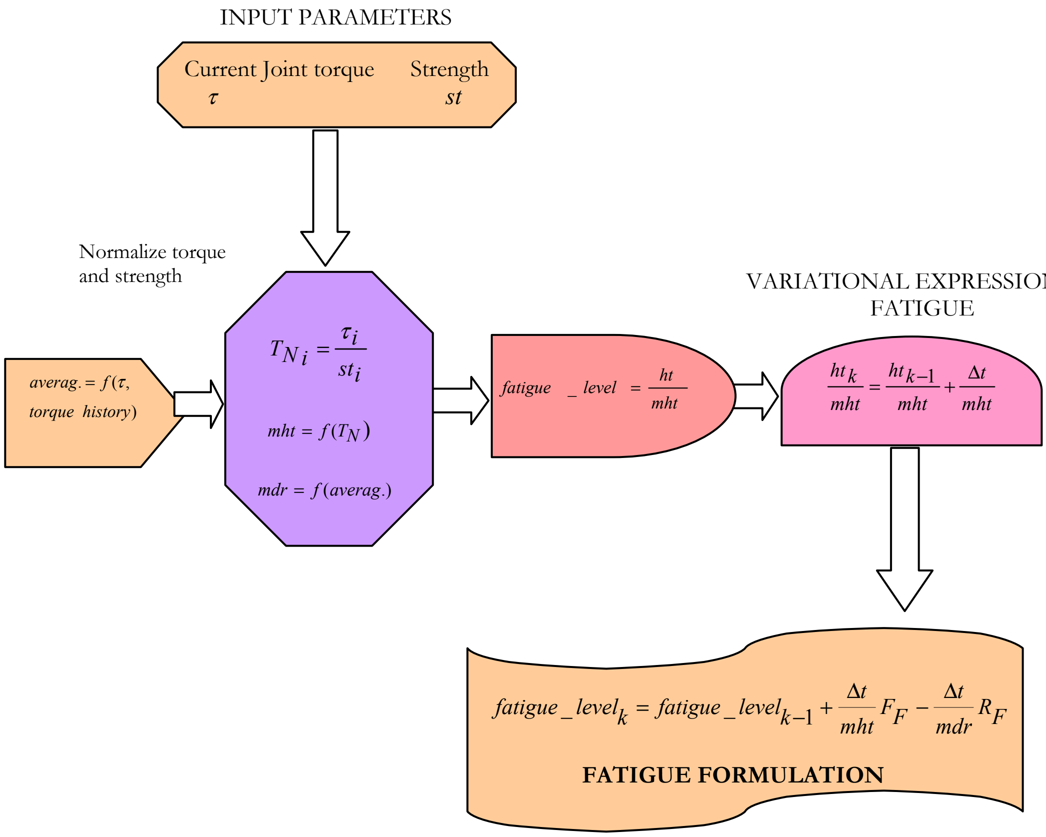


Figure 4-13. Muscular fatigue calculation

4.6 Integrating the Half-joint Concept in the Fatigue Formulation

The fatigue model presented in the previous section is designed to be applied to individual joints. Figure 4-14 shows how each joint is split in a pair of half-joints. The left side of the figure shows a simple hierarchy representing a human arm that contains only shoulder flexion and elbow flexion joints. Then, the hierarchy is split in pairs of half-joints. They are represented by agonist and antagonist joints.

Therefore, our fatigue model is applied to a hierarchy as the one on right side of the arrow.

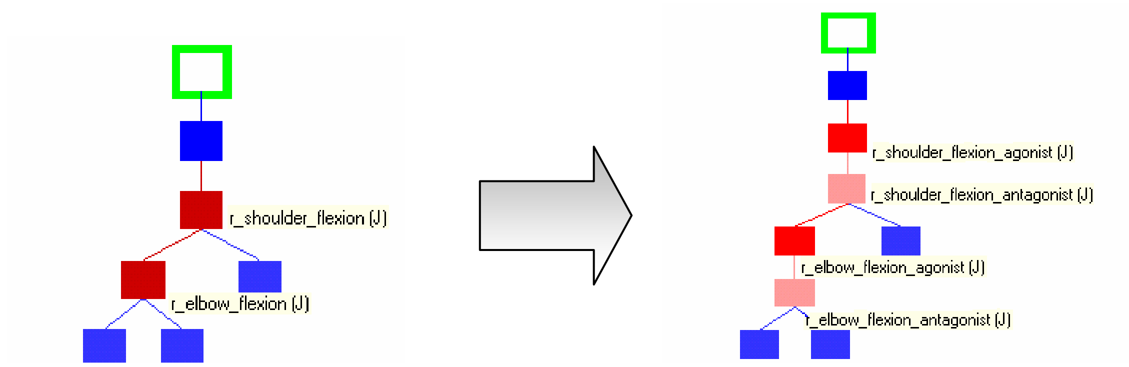


Figure 4-14. Splitting of human arm joints, agonist (red) antagonist (pink)

Fatigue calculation is performed separately for each half-joint of a pair. Figure 4-15 shows an example where the agonist is the active muscle group and the antagonist is the inactive muscle group. Then, the recovery factor R_F described in section 4.5 is null for the active muscle group while the fatigue factor F_F is null for the inactive muscle group. Both muscle groups act always in opposite directions but a same group of muscles (e.g. biceps) acts as active in some cases and as inactive in others.

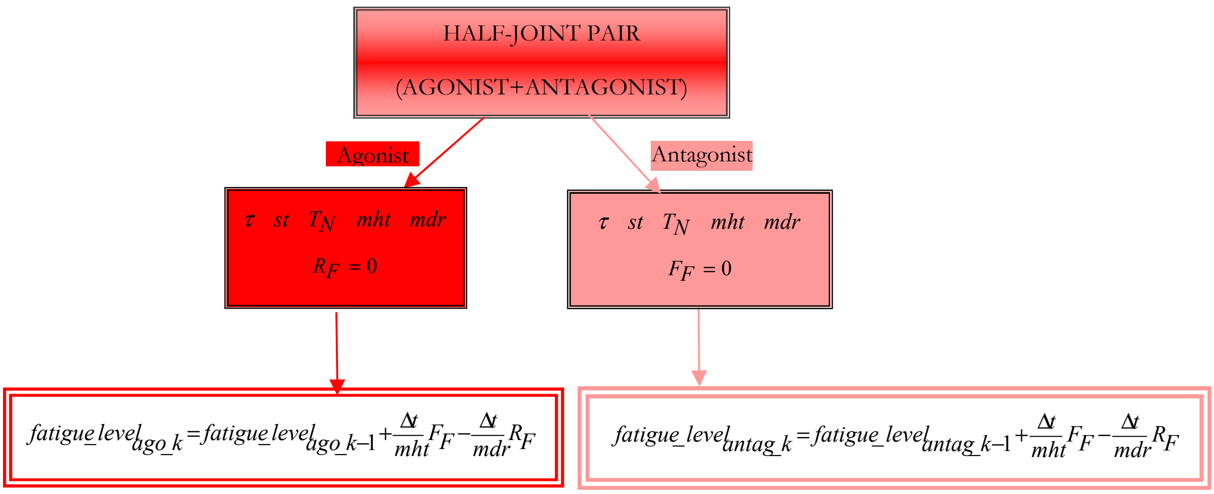


Figure 4-15. An example of fatigue calculation for each joint of a pair

4.7 Summary

In this chapter, we have presented the modeling and evaluation of a physiological factor such as human fatigue at joint level. We describe a muscular fatigue model to be applied to slowly varying human postures. It is mainly based on values of current torque and strength at joint level at each step of time.

The half-joint concept has been introduced and used to calculate fatigue independently for antagonist muscle groups. Depending on the muscle group that actuates as agonist and antagonist, null fatigue and recovery factors are applied.

Unlike other researches that consider fatigue at individual muscle level, we have modeled fatigue at joint level, therefore simplifying and presenting a more general approach.