THE DISTRIBUTION OF THE GALOIS ORBITS OF ALGEBRAIC POINTS

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Lectures 1 & 2: Potential theory on the complex plane

In these lectures, I provided an introduction to logarithmic potential theory on the complex plane, emphasizing its applications to the study of polynomials.

The following is the list of the notions and subjects covered during these lectures:

(1) transfinite diameter, capacity and Chebysheb constant of a compact set;
(2) Fekete points, Fekete and Chebysheb polynomials;
(3) energy and potential of a measure, equilibrium measure;
(4) Frostman theorem on the conductor potential of a compact set;
(5) the fundamental theorem of logarithmic capacity theory;
(6) approximation of compact sets by lemniscates.

References

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[Saf10] E. B. Saff, *Logarithmic potential theory with applications to approximation theory*, Surv. Approx. Theory **5** (2010), 165-200.

Lecture 3: The Fekete-Szegö theorem

The Fekete-Szegö theorem characterizes compact subsets of the complex plane containing an infinite number of Galois orbits of algebraic integers, in terms of its capacity. This key result has inspired a great deal of generalizations, mainly by Cantor and Rumely, to algebraic curves and adelic subsets.

These are the subjects treated in this lecture:

(1) statement of the Fekete and of the Fekete-Szegö theorem;
(2) discriminant of an algebraic integer and proof of the Fekete theorem;
(3) lemniscates defined over the integers and proof of the Fekete-Szegö theorem.

References

[Aut17] P. Autissier, *Autour du théorème de Fekete-Szegö*, École d'été de géométrie d'Arakelov, Grenoble, 2017. Téléchargeable sur `https://www.math.u-bordeaux.fr/~pautissi/`.
[FS55] M. Fekete, G. Szegö, *On algebraic equations with integral coefficients whose roots belong to a given point set*, Math. Z. **63** (1955), 158-172.
[FS55] R. Rumely, *Capacity theory with local rationality. The strong Fekete-Szegö theorem on curves*, Math. Surveys Monogr. Vol. 193, Amer. Math. Soc., 2013.

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Lecture 4: Equidistribution of Galois orbits of algebraic points of small height

The limit distribution of the Galois orbits of generic sequences of points of small height is a central subject in arithmetic geometry.

Here is the list of notions and results covered in this lecture:

(1) height of a rational number;
(2) metrized line bundle on an algebraic variety over the rationals and its associated height function, in the context of Arakelov geometry;
(3) essential minimum of a height function
(4) Yuan's equidistribution theorem for the Galois orbits of generic sequence points of height approaching the essential minimum in the quasi-canonical case;
(5) Mahler measure of a polynomial with complex coefficients, Weil height of an algebraic point in the projective line, the Lehmer problem;
(6) Bilu's equidistribution theorem;
(7) Arakelov heights on the projective line, a counter-example to the equidistribution in the non-quasi-canonical case.

References

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[Yua08] X. Yuan, *Big line bundles over arithmetic varieties*, Invent. Math. **173** (2008), 603-649.

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