

Basic Quantum Mechanics for Mathematicians
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The proposed series of lectures is an introduction into the quantum theory for mathematicians that have little background in Physics but possess certain familiarity with Functional Analysis, Algebra, Dynamical Systems in general and Hamiltonian Dynamics in particular. We will see how the choices made in construction of quantum mechanical models arise from mathematical viewpoint. Then we will concentrate our attention on finite dimensional quantum models: their definition, properties and perturbation methods. The following topics are to be covered in the lectures.

1. A brief introduction into the origins of quantum mechanics.
2. Observables and states in classical mechanics versus observables and states in quantum mechanics: algebras of observables, Poisson brackets, pure and mixed states, evolution operator.
3. Finite-dimensional quantum systems:
 - (a) algebra of Hermitian linear operators on $\mathbb{C}^n$: spectral theory, trace, quantum Poisson bracket
 - (b) States in quantum mechanics: expected value for an observable, density matrix, positive functionals, functions of an observable, probability measures for an observable
 - (c) Standard deviation and expectation of an observable, pure and mixed states, density matrix of a pure state
 - (d) Heisenberg uncertainty relations
 - (e) Quantum Dynamics: Heisenberg picture of quantum dynamics, evolution operator, Schrödinger picture of quantum dynamics
 - (f) Schrödinger equation: derivation, general solution, stationary states. Physical interpretations and limitations of the finite-dimensional theory.
4. Particle with a spin: a non-autonomous quantum system in $\mathbb{C}^2$. Adiabatic quantum theory and classical singular perturbation theory, non-preservation of a normally elliptic slow manifold and quantum transitions, divergent series and exponential asymptotics.
5. Quantum mechanics on infinite-dimensional Hilbert spaces: limitations of the finite-dimensional theory, general theory, quantisation rules, examples. Ehrenfest equation. Connection to the operator theory, Weyl sequences.